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The background model of *CELESTINA*

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Abstract

We describe the background model implemented in *CELESTINA*.

1 Introduction

The current *CELESTINA* version provides two background estimation methods intended to produce skyplots. A first one is based in the proyection of the events with a reconstructed incident direction located in the half of the camera where th anti-source is located in the other half of the camera. A second one is the so-called **background model** which does not require to know in advance the source and anti-source positions, but only provides a good estimate for point like sources. This later method is explained in this note, emphasizing the formal description of the first step of the method, which consists in a linear transformation as we show below.

2 Notation

In this note all the spatial coordinates are in the camera reference frame, where the coordinates of the camera center are given by $0, 0$. The rate per unit area of the gamma source and the background at position x, y are denotated as $S(x, y)$ and $B(x, y)$ respectively. The total rate per unit area after all analysis cuts is denotated as $M(x, y) = S(x, y) + B(x, y)$. We use the notation $G(x, y|\sigma)$ for a the value of a bi-dimensional gaussian distribution centered at $0, 0$, of width σ evaluated at x, y. The PSF of the telescope is called σ_{PSF}. The unknown position of the gamma-ray source is x_S, y_S. We denote the convolution operation of a function $f(x, y)$ with a kernel $K(x, y)$ as

$$K(x, y) * f(x, y) = \int dx'dy' f(x', y') \times K(x' - x, y' - y) \tag{1}$$

3 Background estimate construction

In a firts step, the background model estimate is given by

$$B^*(x, y) = 2 \times G(x, y|\sqrt{2}\sigma_B) * M(x, y) - G(x, y|\sigma_B) * M(x, y) \tag{2}$$

where σ_B is a model parameter and the superindice $*$ denotates estimate.

In a second step a further correction of the radial distortions on this estimate is performed by scaling it with a function depending of the distance to the camera center obtained by fitting

$$\Delta(r) = \frac{\int dx'dy' M(x', y')\delta(x'^2 + y'^2 - r^2)}{\int dx'dy' B^*(x', y')\delta(x'^2 + y'^2 - r^2)} \tag{3}$$

where δ is a Dirac delta distribution. The fit function is an expansion in a normalized radial basis function space with typically 10 dimensions, although for large statistics this number can be increase. The fit procedure is a robust Least Trimmed Squares fit using a minimal fraction of the 90% of the points where , which guarantees that the correction factor is not overestimated in the case a gamma-ray source is in the field of view. Figure 1 shows the typical fit to the function $\Delta(r)$ in a case without any signal, showing that it is close to 1 for $r < 2°$, and the correction is maximum at camera center.

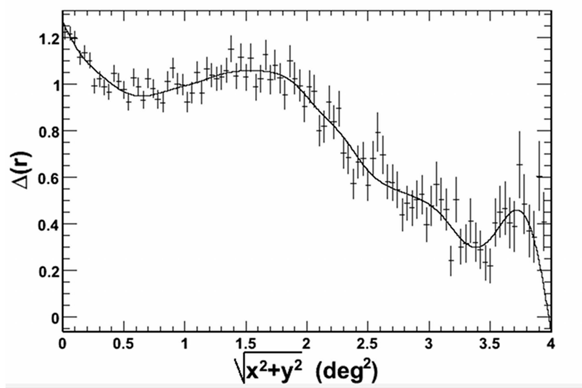

Figure 1: Fit of the correction factor $\Delta(r)$.

4 Formal justification

In this section we formally demostrate tha eq. 2 is a good estimate for the background. To this end we parameterize $B(x, y)$ as

$$B(x,y) = \sum_{i=1}^{N} b_i \times G(x - x_i, y - y_i|\sigma_i^B) \tag{4}$$

where N can be infinity and all $b_i > 0$. Notice that this parameterization is always possible since the set of all gaussians form an overcomplete basis. In addition we assume that the following approximation holds

$$S(x,y) = s \times G(x - x_S, y - y_s|\sigma_{PSF}) \tag{5}$$

where s is the number of gamma-rays detected from the source. This approximation assumes that the source is point-like and that no de-rotation is performed, which is the worst case for background estimation because the signal is concentrated in single point. Nevertheless all the results obtained under this approximation hold if $S(x, y)$ can be parameterized as $B(x, y)$ in equation 4 with the condition that the corresponding x_{Si}, y_{Si} have a zero-measure distribution due to the fact that the transformation given by 2 is a linear one.

With these assumption, the estimate given by equation 2 is obtained by sampling from the distribution given by

$$\begin{aligned} B^*(x,y) = \quad & 2 \times G(\sqrt{2}\sigma_B) * (B(x,y) + S(x,y)) - G(\sigma_B) * (B(x,y) + S(x,y)) = \\ & \left(2 \sum_{i=1}^{N} b_i \times G\left(x - x_i, y - y_i|\sqrt{(\sigma_i^B)^2 + 2\sigma_B^2}\right) - b_i \times G\left(x - x_i, y - y_i|\sqrt{(\sigma_i^B)^2 + \sigma_B^2}\right)\right) + \\ & \left(2s \times G\left(x - x_S, y - y_S|\sqrt{\sigma_{PSF}^2 + 2\sigma_B^2}\right) - s \times G\left(x - x_S, y - y_S|\sqrt{\sigma_{PSF}^2 + \sigma_B^2}\right)\right) \end{aligned} \tag{6}$$

By choosing a value of σ_B such that

$$(\sigma_i^B)^2 \gg \sigma_B^2 \tag{7}$$

for any i, and thus

$$\sqrt{(\sigma_i^B)^2 + \sigma_B^2} = \sigma_B^i \times (1 + O(\frac{\sigma^B}{(\sigma_i^B)^2})) \tag{8}$$

In consequence we have that eq. 6 can be writen as

$$\begin{aligned} B^*(x,y) \simeq \underbrace{\sum_{i=1}^{N} b_i \times G\left(x - x_i, y - y_i|\sigma_i^B\right)}_{B(x,y)} + O(\frac{\sigma_B^2}{(\sigma_i^B)^2}) + \\ + \underbrace{2s \times G\left(x - x_S, y - y_S|\sqrt{\sigma_{PSF}^2 + 2\sigma_B^2}\right) - s \times G\left(x - x_S, y - y_S|\sqrt{\sigma_{PSF}^2 + \sigma_B^2}\right)}_{S^*(x,y)} \end{aligned} \tag{9}$$

where we have defined the function S^* as the addition of the two last two terms on the right hand side of the equation, and the term $O(\epsilon)$ means a function of order ϵ.

With this, the estimate signal given eq. 9 is

$$M(x,y) - B^*(x,y) = S(x,y) - S^*(x,y) + O(\frac{\sigma_B^2}{(\sigma_i^B)^2}) \tag{10}$$

A plot of the function $S(x,y) - S^*(x,y)$ scaled such that $S(x_S, y_S) = 1$ is shown in figure 2. It shows that for σ_B large enough it is a good approximation for $S(x,y)$. Figure 3 shows these two functions for the case $\sigma_B = \sigma_{PSF}$, which is the one chosen for the MAGIC-I background model. Moreover, the application of the multiplicative correction factor defined in eq. 3, by computing $M(x,y) - \Delta(\sqrt{x^2+y^2}) \times B^*(x,y)$ instead of eq. 10 guarantees a better approximation if no gamma-rays source is in the field of view by construction, and in the the case of the presence of one it applies a further suppression factor to $S^*(x,y)$ due to the Least Trimmed Squares fit procedure used.

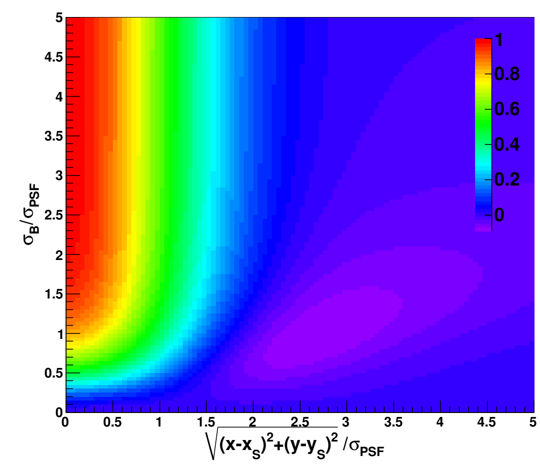

Figure 2: Plot of the function $(S(x,y) - S^*(x,y))2\pi/s$ as a function to the distance to x_S, y_S and the ratio σ_B/σ_{PSF}.

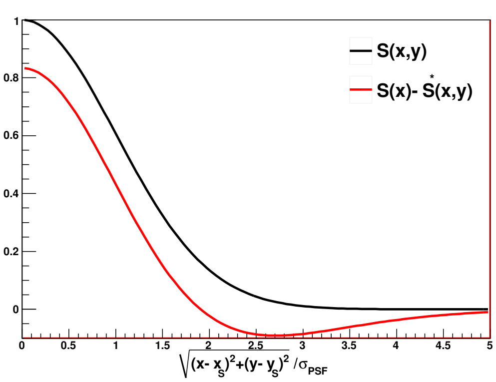

Figure 3: Plot of the functions $S^*(x,y) = (S(x,y) - S^*(x,y))2\pi/s$ and $S(x,y)2\pi/s$ as a function to the distance to x_S, y_S for the parameter condition $\sigma_B = \sigma_{PSF}$.

5 Conclusion

As a conclusion, we can say that $M(x,y) - \Delta(\sqrt{x^2+y^2}) \times B^*(x,y)$ is an approximation for $S(x,y)$, that in the case of the parameter σ_B being equal to σ_{PSF} underestimates the true value by less than 20% of $S(x_S, y_S)$ close to the point x_S, y_S, and far from it it approximates to the true value $B(x,y)$, this approximation being better that of the order of σ_B^2/σ_i^B due to the $\Delta(r)$ correction factor. This makes this background approximation useful to search for serendipitous sources in the field of view of MAGIC-I, although it is not well suited to provide background estimates useful for flux computations. It remains to show that the condition given by eq.

7 holds. We have checked this experimentally by explicitly searching for a parameterization as eq. 4, finding that taking $\sigma_i \simeq 0.3^\circ$ for all i such construction is possible giving good residuals. Taking into account that $\sigma_{PSF} \simeq 0.1^\circ$ above 150 GeV, the value of $\sigma_B = \sigma_{PSF}$ is a good compromise between the condition of eq. 7 and the necessity of taking a large enough value for the parameter as illustrated by figure 2.