TRANSFORMATIONS BETWEEN THE SKY, THE LOCAL AND THE CAMERA REFERENCE SYSTEMS
(*Formulas used in the C^{++} class MStarCamTrans*)

W. Wittek
wittek@mppmu.mpg.de

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Abstract

With a given orientation of the telescope, each point in the camera corresponds to a certain direction in local coordinates and to a certain direction in sky coordinates. This note deals with the transformations between the different coordinate systems.

Contents

1 Introduction

A direction in space can be specified in various reference systems. In an IACT experiment the systems of interest are the sky, the local and the camera coordinate systems. For a fixed orientation of the telescope axis, there are unique relations between the coordinates in the different system. These relations are the topic of this note.

The outline of the note is as follows : In Section 2 the different coordinate systems are defined. The transformations between the various systems are derived in Sections 3, 4 and 5. In Section 6, the orientation of the telescope axis is calculated from the coordinates in the camera system and the corresponding local (or sky) coordinates of a direction in space. The calculation of the coordinates of a second direction, when the coordinates of another direction are given, is described in Section 7. How the various transformations can be used to draw grids of local and/or sky coordinates in the camera is described in Section 8. In Section 9 some special transformations are discussed which might be useful for the wobble mode of data taking.

All transformations described in this note are implemented as member functions of the C^{++} class $MStarCamTrans$.

A word of warning : The transformations between local and sky coordinates as described in this note are only approximate. The corresponding member functions of $MStarCamTrans$ are $LocToCel$ and $CelToLoc$. All approximations can be avoided by replacing these functions by the exact transformations.

2 Definition of the coordinate systems

Three coordinate systems will be considered :

- System A : A local coordinate system (x_A, y_A, z_A), in which the **local (zenith and azimuthal) angles** (θ, φ) are defined.

- System B : An equatorial coordinate system (x_B, y_B, z_B), in which the **sky coordinates** (δ, ϕ), i.e declination and the hour angle, are defined. ϕ is defined to be zero when the source is culminating at the geographical longitude of the telescope. The relation between the hour angle ϕ and the right ascension RA is

$$\phi \;=\; -RA \;+\; c_0 \;+\; c_1 \cdot t \tag{1}$$

  where c_0 andc_1 are independent of the source and independent of the time t with $c_1 \;=\; \frac{366,25}{365,25} \cdot \frac{360^\circ}{24\ \mathrm{h}}$. If RA and ϕ are known for an arbitrary point on the sky at the time t the constant c_0 can be determined and the relation between RA and ϕ is known for all sky directions, at all times.

- System T : A telescope coordinate system (x_T, y_T, z_T), in which the **camera coordinates** $(x_C,\ y_C)$ are defined.

The definition of the systems A and B is given in [1] and it can also be seen from Figs. 1 and 2. The vectors $\vec{a} = (a_1,\ 0,\ a_3)$, $\vec{z}$ and $\vec{r}$ denote the earth-rotation axis (pointing to the celestial north pole), the zenith direction and the position of a point on the sky respectively. For La Palma, with a geographical latitude of $Lat = 28.8^\circ$, $a_1 = \cos(Lat)$ and $a_3 = -\sin(Lat)$ are equal to 0.8763 and -0.4818 respectively.

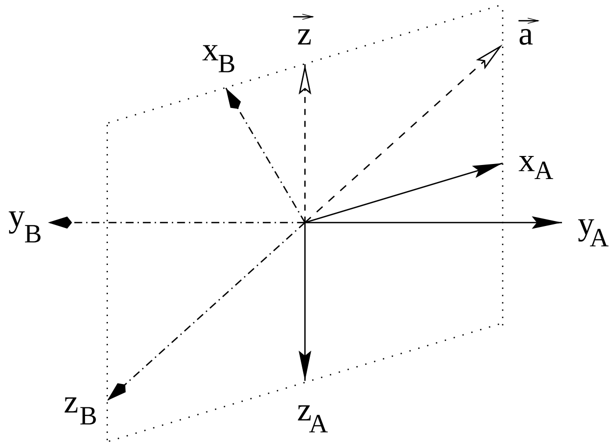

Figure 1: Directions of the axes (x_A, y_A, z_A) and (x_B, y_B, z_B) of the systems A and B respectively. The plane formed by the earth-rotation axis $\vec{a}$ and the zenith direction $\vec{z}$ contains also the directions x_A, z_A and x_B, z_B. The directions y_A and y_B are perpendicular to this plane. The zenith direction $\vec{z}$ is opposite to z_A, and $\vec{a}$ is opposite to z_B.

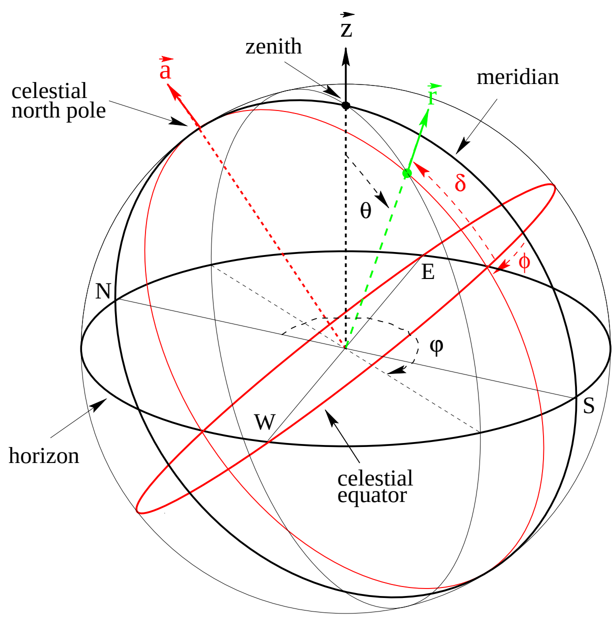

Figure 2: Definition of the zenith angle θ, the azimuthal angle φ, the declination δ and the hour angle ϕ. The vectors $\vec{a}, \vec{z}$ and $\vec{r}$ denote the earth-rotation axis (pointing to the celestial north pole), the zenith direction and the position of a point on the sky respectively.

The telescope system T is defined as :

- x-axis in the direction of $\vec{e}_x \; = \; \dfrac{\vec{r}_0 \times \vec{z}}{|\vec{r}_0 \times \vec{z}|}$
- y-axis in the direction of $\vec{e}_y \; = \; \vec{e}_x \times \vec{r}_0$
- z-axis in the direction of $\vec{e}_z \; = \; -\vec{r}_0$

where $\vec{r}_0$ represents the direction the telescope is pointing to.

For understanding the definition of the telescope system the following consideration may be useful : When the observer is looking from the center of the reflector in the direction of the telescope axis (towards the camera) the x_T-axis is pointing horizontally to the right, the y_T-axis upwards and the z_T-axis towards the observer.

The origin of all three systems is assumed to be in the center of the reflector.

A natural definition of the camera plane is the plane perpendicular to $\vec{e}_z$ at $z_T \; = \; -R_C$, where R_C is the distance of the camera from the reflector center. For convenience, a **fictive camera system** (x_C, y_C) is defined in the plane at $z_T \; = \; -1$ with its center on the $\vec{e}_z$ axis. With this definition small x_C and small y_C near $(x_C, y_C) = (0,0)$ in the fictive camera system correspond directly to angles in the sky (see eqs. (34) and (37)) :

$$\Delta x_C \; = \; -\sin\theta_0 \cdot \Delta \tan(\varphi - \varphi_0) \; \simeq \; -\sin\theta_0 \cdot \Delta\varphi \tag{2}$$

$$\Delta y_C \; = \; \Delta \tan(\theta - \theta_0) \; \simeq \; \Delta\theta \tag{3}$$

The position (x_{trueC}, y_{trueC}) in the true camera plane is obtained from (x_C, y_C) by

$$x_{trueC} \; = \; R_C \cdot x_C \qquad y_{trueC} \; = \; R_C \cdot y_C \tag{4}$$

By definition, the center of the camera $(x_C, y_C) = (0,0)$ corresponds to the direction (θ_0, φ_0) (or (δ_0, ϕ_0)) the telescope is pointing to.

3 Transformations between the local and the equatorial system

According to [1] the transformations between the local and the sky system read :

$$\vec{r}_A \; = \; \begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix} \; = \; \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ -\cos\theta \end{pmatrix} \; = \; \begin{pmatrix} a_3\cos\delta\cos\phi \; + \; a_1\sin\delta \\ -\cos\delta\sin\phi \\ -a_1\cos\delta\cos\phi \; + \; a_3\sin\delta \end{pmatrix} \tag{5}$$

$$\vec{r}_B \; = \; \begin{pmatrix} x_B \\ y_B \\ z_B \end{pmatrix} \; = \; \begin{pmatrix} \cos\delta\cos\phi \\ \cos\delta\sin\phi \\ -\sin\delta \end{pmatrix} \; = \; \begin{pmatrix} a_3\sin\theta\cos\varphi \; + \; a_1\cos\theta \\ -\sin\theta\sin\varphi \\ -a_1\sin\theta\cos\varphi \; + \; a_3\cos\theta \end{pmatrix} \tag{6}$$

or

$$\theta \; = \; \arccos(a_1 \cdot x_B + a_3 \cdot z_B) \qquad \varphi \; = \; \arctan(-y_B, \; a_3 \cdot x_B - a_1 \cdot z_B) \tag{7}$$

$$\delta \; = \; \arcsin(a_1 \cdot x_A + a_3 \cdot z_A) \qquad \phi \; = \; \arctan(-y_A, \; a_3 \cdot x_A - a_1 \cdot z_A) \tag{8}$$

The transformations are programmed in the functions *CelToLoc* and *LocToCel* respectively.

It should be noted that these transformations are only approximate, because they do not take into account effects due to precession, nutation and refraction.

By precession one understands the rotation of the earth rotation axis, on a circle of radius $23°27'$, around the axis of the ecliptic. The period of this rotation is 25700 years. Precession causes a change of RA in the order of 0.1 time minutes per year, and a change of δ in the order of 0.2 arc minutes per year [2].

Superimposed on the precession is another rotation, the nutation, with a period of 19 years and a smaller amplitude than that of the precession.

Because of refraction, a light ray when passing through the atmosphere is bent such that a source appears under a higher elevation. The change in elevation is 0", 33", 1'09", 2'37" and 34'50" for zenith angles of 0°, 30°, 50°, 70° and 90° respectively [2].

4 Transformations between the local and the camera system at fixed telescope orientation

4.1 Transformation from local coordinates (θ, φ) to coordinates (x_C, y_C) in the fictive camera system, when the telescope orientation is fixed at $(\theta_0,\ \varphi_0)$

The representations of the vectors $\vec{e}_x,\ \vec{e}_y,\ \vec{e}_z,\ \vec{r}_0$ and of some sky direction $\vec{r}^{\,orig}$ in system A are

$$\vec{e}_{x,A} = \begin{pmatrix} -\sin\varphi_0 \\ \cos\varphi_0 \\ 0 \end{pmatrix} \qquad \vec{e}_{y,A} = \begin{pmatrix} -\cos\theta_0\cdot\cos\varphi_0 \\ -\cos\theta_0\cdot\sin\varphi_0 \\ -\sin\theta_0 \end{pmatrix} \tag{9}$$

$$\vec{r}_{0,A} = \begin{pmatrix} \sin\theta_0\cdot\cos\varphi_0 \\ \sin\theta_0\cdot\sin\varphi_0 \\ -\cos\theta_0 \end{pmatrix} = -\vec{e}_{z,A} \qquad \vec{r}_A^{\,orig} = \begin{pmatrix} \sin\theta\cdot\cos\varphi \\ \sin\theta\cdot\sin\varphi \\ -\cos\theta \end{pmatrix} \tag{10}$$

It follows that the components of $\vec{r}^{\,orig}$ in the telescope system are

$$\vec{r}_T^{\,orig} = \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} = \begin{pmatrix} \vec{e}_{x,A}\cdot\vec{r}_A^{\,orig} \\ \vec{e}_{y,A}\cdot\vec{r}_A^{\,orig} \\ \vec{e}_{z,A}\cdot\vec{r}_A^{\,orig} \end{pmatrix} = M\cdot\vec{r}_A^{\,orig} \tag{11}$$

with

$$M = \begin{pmatrix} -\sin\varphi_0 & \cos\varphi_0 & 0 \\ -\cos\theta_0\cos\varphi_0 & -\cos\theta_0\sin\varphi_0 & -\sin\theta_0 \\ -\sin\theta_0\cos\varphi_0 & -\sin\theta_0\sin\varphi_0 & \cos\theta_0 \end{pmatrix} \tag{12}$$

Thus

$$\vec{r}_T^{\,orig} = \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} = \begin{pmatrix} \sin\theta\cdot\sin(\varphi-\varphi_0) \\ \sin\theta_0\cos\theta - \cos\theta_0\cdot\sin\theta\cdot\cos(\varphi-\varphi_0) \\ -\cos\theta_0\cos\theta - \sin\theta_0\cdot\sin\theta\cdot\cos(\varphi-\varphi_0) \end{pmatrix} \tag{13}$$

Note that these are the components of the original direction $\vec{r}^{\,orig}$ in the telescope system (before the reflection at the reflector). The reflection at the reflector is equivalent to a rotation of $\vec{r}^{\,orig}$ by 180° around the telescope orientation $\vec{r}_0$:

$$\vec{r}_T^{\,ref} = \begin{pmatrix} x^{ref} \\ y^{ref} \\ z^{ref} \end{pmatrix} = \begin{pmatrix} -x_T \\ -y_T \\ z_T \end{pmatrix} = S \cdot \vec{r}_T^{\,orig} \tag{14}$$

with

$$S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{pmatrix} = S^{-1} \tag{15}$$

Using (11) the components of $\vec{r}^{\,ref}$ in the telescope system (after reflection) can be written as

$$\vec{r}_T^{\,ref} = \begin{pmatrix} x^{ref} \\ y^{ref} \\ z^{ref} \end{pmatrix} = S \cdot \vec{r}_T^{\,orig} = S \cdot M \cdot \vec{r}_A^{\,orig} \tag{16}$$

$$= \begin{pmatrix} -\sin\theta \cdot \sin(\varphi - \varphi_0) \\ -\sin\theta_0 \cos\theta + \cos\theta_0 \cdot \sin\theta \cdot \cos(\varphi - \varphi_0) \\ -\cos\theta_0 \cos\theta - \sin\theta_0 \cdot \sin\theta \cdot \cos(\varphi - \varphi_0) \end{pmatrix} \tag{17}$$

$$= \begin{pmatrix} -\sin\theta \cdot \sin(\varphi - \varphi_0) \\ \sin(\theta - \theta_0) - \cos\theta_0 \cdot \sin\theta \cdot [1 - \cos(\varphi - \varphi_0)] \\ -\cos(\theta - \theta_0) + \sin\theta_0 \cdot \sin\theta \cdot [1 - \cos(\varphi - \varphi_0)] \end{pmatrix} \tag{18}$$

The coordinates (x_C, y_C) in the fictive camera system are now obtained by determining the intersection point $\vec{r}_T^{\,cam}$ of the line $(\tau \cdot \vec{r}_T^{\,ref})$ with the plane $z_T = -1$:

$$\vec{r}_T^{\,cam} = \begin{pmatrix} x_C \\ y_C \\ -1 \end{pmatrix} = \frac{-1}{z^{ref}} \cdot \begin{pmatrix} x^{ref} \\ y^{ref} \\ z^{ref} \end{pmatrix} \tag{19}$$

or

$$\begin{pmatrix} x_C \\ \\ y_C \end{pmatrix} = \frac{1}{\sqrt{1 - (x^{ref})^2 - (y^{ref})^2}} \cdot \begin{pmatrix} x^{ref} \\ \\ y^{ref} \end{pmatrix} \tag{20}$$

$$= \begin{pmatrix} \dfrac{-\sin\theta \cdot \sin(\varphi - \varphi_0)}{\cos\theta_0 \cdot \cos\theta + \sin\theta_0 \cdot \sin\theta \cdot \cos(\varphi - \varphi_0)} \\ \\ \dfrac{-\sin\theta_0 \cdot \cos\theta + \cos\theta_0 \cdot \sin\theta \cos(\varphi - \varphi_0)}{\cos\theta_0 \cdot \cos\theta + \sin\theta_0 \cdot \sin\theta \cdot \cos(\varphi - \varphi_0)} \end{pmatrix} \tag{21}$$

$$= \begin{pmatrix} \dfrac{-\sin\theta \cdot \sin(\varphi - \varphi_0)}{\cos(\theta - \theta_0) - \sin\theta_0 \cdot \sin\theta \cdot [1 - \cos(\varphi - \varphi_0)]} \\ \\ \dfrac{\sin(\theta - \theta_0) - \cos\theta_0 \cdot \sin\theta \cdot [1 - \cos(\varphi - \varphi_0)]}{\cos(\theta - \theta_0) - \sin\theta_0 \cdot \sin\theta \cdot [1 - \cos(\varphi - \varphi_0)]} \end{pmatrix} \tag{22}$$

since $\vec{r}_T^{\,ref}$ is a unit vector. Note that the scaling factor $(-1/z^{ref})$ is close to $+1$, because (due to the small field of view of the camera) the directions (θ, φ) and (θ_0, φ_0) are close.

The calculation of $(x_C,\ y_C)$ from $(\theta,\ \varphi)$ and $(\theta_0,\ \varphi_0)$ is programmed in the function *Loc0LocToCam*.

4.2 Transformation from (x_C, y_C) to (θ, φ) at fixed (θ_0, φ_0)

The point (x_C, y_C) in the fictive camera system is the point $(x_C, y_C, -1)$ in the telescope system, from which the unit vector $\vec{r}_T^{\,ref}$ is derived by

$$\vec{r}_T^{\,ref} = \begin{pmatrix} x^{ref} \\ y^{ref} \\ z^{ref} \end{pmatrix} = \frac{1}{\sqrt{1 + x_C^2 + y_C^2}} \cdot \begin{pmatrix} x_C \\ y_C \\ -1 \end{pmatrix} \tag{23}$$

Inverting (16) the original direction $\vec{r}_A^{\,orig}$ in system A is given by

$$\vec{r}_A^{\,orig} = \begin{pmatrix} \sin\theta \cdot \cos\varphi \\ \sin\theta \cdot \sin\varphi \\ -\cos\theta \end{pmatrix} = M^{-1} \cdot S^{-1} \cdot \vec{r}_T^{\,ref} = M^T \cdot S \cdot \vec{r}_T^{\,ref} \tag{24}$$

$$= \begin{pmatrix} \sin\varphi_0 & \cos\theta_0 \cdot \cos\varphi_0 & -\sin\theta_0 \cdot \cos\varphi_0 \\ -\cos\varphi_0 & \cos\theta_0 \cdot \sin\varphi_0 & -\sin\theta_0 \cdot \sin\varphi_0 \\ 0 & \sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} x^{ref} \\ y^{ref} \\ z^{ref} \end{pmatrix} \tag{25}$$

Knowing $\vec{r}_A^{\,orig}$ the angles $(\theta,\ \varphi)$ can be calculated.

$(\theta,\ \varphi)$ can also be obtained by calculating $\sin(\varphi - \varphi_0)$, $\cos(\varphi - \varphi_0)$ and $\tan^2\theta$ from (21)

$$\sin(\varphi - \varphi_0) = \frac{-x_C}{(\cos\theta_0 - y_C \cdot \sin\theta_0) \cdot \tan\theta} \tag{26}$$

$$\cos(\varphi - \varphi_0) = \frac{\sin\theta_0 + y_C \cdot \cos\theta_0}{(\cos\theta_0 - y_C \cdot \sin\theta_0) \cdot \tan\theta} \tag{27}$$

$$\tan^2\theta = \frac{(\sin\theta_0 + y_C \cdot \cos\theta_0)^2 + x_C^2}{(\cos\theta_0 - y_C \cdot \sin\theta_0)^2} \tag{28}$$

and converting the trigonometric functions into the angles. The ambiguity in the sign of $\cos\theta$ is resolved in the following way : If θ_0 is close to 0 or π choose that θ which is closer to θ_0, i.e. require the same sign for $\cos\theta$ and $\cos\theta_0$. Otherwise choose that θ which is compatible with a small difference $|\varphi - \varphi_0|$, i.e. $\cos(\varphi - \varphi_0) > 0$.

The calculation of $(\theta,\ \varphi)$ from $(x_C,\ y_C)$ and $(\theta_0,\ \varphi_0)$ is programmed in the function *Loc0CamToLoc*.

5 Transformations between the sky and the camera system at fixed telescope orientation

5.1 Transformation from sky coordinates (δ, ϕ) to coordinates (x_C, y_C) in the fictive camera system, when the telescope orientation is fixed at $(\delta_0,\ \phi_0)$

The transformation is obtained by transforming the directions $(\delta_0,\ \phi_0)$ and (δ, ϕ) into local coordinates $(\theta_0,\ \varphi_0)$ and $(\theta,\ \varphi)$ and then applying the transformation $Loc0LocToCam$.

The calculation of $(x_C,\ y_C)$ from $(\delta,\ \phi)$ and $(\delta_0,\ \phi_0)$ is programmed in the function $Cel0CelToCam$.

5.2 Transformation from (x_C, y_C) to (δ, ϕ), at fixed $(\delta_0,\ \phi_0)$

The transformation is obtained by transforming the direction $(\delta_0,\ \phi_0)$ into local coordinates $(\theta_0,\ \varphi_0)$, applying the transformation $Loc0CamToLoc$ and then transforming the direction $(\theta,\ \varphi)$ into the sky coordinates $(\delta,\ \phi)$.

The calculation of $(\delta,\ \phi)$ from $(x_C,\ y_C)$ and $(\delta_0,\ \phi_0)$ is programmed in the function $Cel0CamToCel$.

6 Calculation of the telescope orientation

6.1 Calculation of the telescope orientation (θ_0, φ_0) from (x_C, y_C) and (θ, φ)

Solving (28) for $\tan^2\theta_0$ yields

$$\tan\theta_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{29}$$

with

$$a = 1 + x_C^2 - y_C^2 \cdot \tan^2\theta \tag{30}$$
$$b = 2 \cdot y_C \cdot [1 + \tan^2\theta] \tag{31}$$
$$c = x_C^2 + y_C^2 - \tan^2\theta \tag{32}$$

The sign in (29) is chosen as follows : If θ is close to 0 or π choose that θ_0 which is closer to θ, i.e. require the same sign for $\cos\theta$ and $\cos\theta_0$. Otherwise choose that θ which is compatible with a small difference $|\varphi - \varphi_0|$, i.e. $\cos(\varphi - \varphi_0) > 0$. By definition, $\sin\theta_0$ is always > 0.

Knowing θ_0, $\varphi - \varphi_0$ can be calculated using (26) and (27).

The calculation of $(\theta_0,\ \varphi_0)$ from $(x_C,\ y_C)$ and $(\theta,\ \varphi)$ is programmed in the function $LocCamToLoc0$.

6.2 Calculation of the telescope orientation (δ_0, ϕ_0) from (x_C, y_C) and (δ, ϕ)

The transformation is obtained by transforming the direction (δ, ϕ) into local coordinates $(\theta,\ \varphi)$, applying the transformation $LocCamToLoc0$ and then transforming the direction $(\theta_0,\ \varphi_0)$ into sky coordinates (δ_0, ϕ_0).

The calculation of $(\delta_0,\ \phi_0)$ from $(x_C,\ y_C)$ and $(\delta,\ \phi)$ is programmed in the function $CelCamToCel0$.

7 Calculation of the coordinates of a second direction if the coordinates of another direction are given

7.1 Calculation of the local coordinates (θ_2, φ_2) for a given point (x_2, y_2) in the camera if the local coordinates (θ_1, φ_1) and camera coordinates (x_1, y_1) are given for another direction

This transformation, which is programmed in the member function $LocCamCamToLoc$, is obtained by applying one after another the transformations $LocCamToLoc0$ and $Loc0CamToLoc$.

7.2 Calculation of the point (x_2, y_2) in the camera for a given local direction (θ_2, φ_2) if the local coordinates (θ_1, φ_1) and camera coordinates (x_1, y_1) are given for another direction

This transformation, which is programmed in the member function $LocCamLocToCam$, is obtained by applying one after another the transformations $LocCamToLoc0$ and $Loc0LocToCam$.

7.3 Calculation of the sky coordinates (δ_2, ϕ_2) for a given point (x_2, y_2) in the camera if the sky coordinates (δ_1, ϕ_1) and camera coordinates (x_1, y_1) are given for another direction

This transformation, which is programmed in the member function $CelCamCamToCel$, is obtained by applying one after another the transformations $CelCamToCel0$ and $Cel0CamToCel$.

7.4 Calculation of the point (x_2, y_2) in the camera for a given sky direction (δ_2, ϕ_2) if the sky coordinates (δ_1, ϕ_1) and camera coordinates (x_1, y_1) are given for another direction

This transformation, which is programmed in the member function $CelCamCelToCam$, is obtained by applying one after another the transformations $CelCamToCel0$ and $Cel0CelToCam$.

8 Drawing grids of local and sky coordinates in the camera

In the previous sections all tools have been provided to draw lines of constant θ, or constant φ or constant δ or constant ϕ in the camera.

The member functions plotting the grids are called $PlotGridAtDec0H0$, $PlotGridAtTheta0Phi0$. The local or sky coordinates of the camera center, which by definition describe the orientation of the telescope, have to be given as arguments, respectively. Knowing the coordinates of a direction corresponding to the center of the camera the coordinates in the camera of any other direction can be calculated using the functions $Loc0LocToCam$ and $Cel0CelToCam$ respectively.

9 Some special cases

If a source direction (x_C, y_C) is given in the fictive camera system, equations (26), (27), (28) and (29) may be used

- to determine the original source direction (θ, φ) for a fixed telescope orientation (θ_0, φ_0), or

- to calculate the telescope direction (θ_0, φ_0) for a fixed source direction (θ, φ)

9.1 Transformations for $(x_C,\ y_C) = (x_C,\ 0)$

For a given telescope orientation (θ_0, φ_0), a source position $(x_C, y_C) = (x_C, 0)$ on the x-axis of the fictive camera system corresponds to an original source direction (θ, φ), where θ and φ are given by

$$\cos\theta = \frac{\cos\theta_0}{\sqrt{1 + x_C^2}} \tag{33}$$

$$\tan(\varphi - \varphi_0) = \frac{-x_C}{\sin\theta_0} \tag{34}$$

By inverting this equation one obtains for a given source direction (θ, φ) with the position $(x_C, 0)$ in the fictive camera system the telescope orientation (θ_0, φ_0) by

$$\cos\theta_0 = \cos\theta \cdot \sqrt{1 + x_C^2} \tag{35}$$

$$\tan(\varphi - \varphi_0) = \frac{-x_C}{\sqrt{\sin^2\theta - x_C^2 \cos^2\theta}} \tag{36}$$

9.2 Transformations for $(x_C,\ y_C) = (0,\ y_C)$

For a given telescope orientation (θ_0, φ_0), a source position $(x_C, y_C) = (0, y_C)$ on the y-axis of the fictive camera system corresponds to a direction (θ, φ), where θ and φ are given by

$$\tan(\theta - \theta_0) = y_C \tag{37}$$

$$\varphi = \varphi_0 \tag{38}$$

By inverting this equation one obtains for a given source direction (θ, φ) with the position $(0, y_C)$ in the fictive camera system the telescope orientation (θ_0, φ_0) by

$$\tan(\theta - \theta_0) = y_C \tag{39}$$

$$\varphi_0 = \varphi \tag{40}$$

References

[1] W. Wittek, MAGIC-TDAS 00-11 (2000)

[2] A. Unsöld and B. Baschek, "Der Neue Kosmos", Springer-Verlag (1999)