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Definition of the Angular Resolution

J. Cortina

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Abstract

I define the angular resolution following the Rayleigh criterion that is commonly used among astronomers. I also relate most of the parameters that are used in reference to the Point Spread Function with the angular resolution and make a final consideration about the source position accuracy.

Contents

1 Point Spread Function

A point-like source generates a distribution of photons at the focal plane of a telescope. This distribution is called Point Spread Function (PSF).

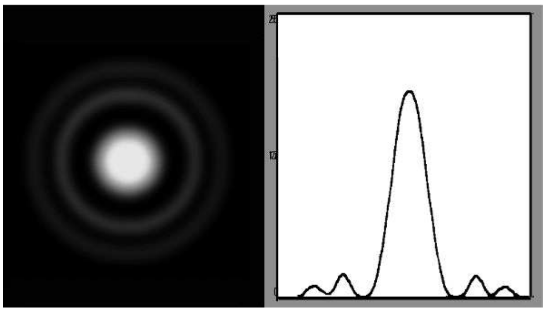

Figure 1: *Left: photon distribution in the focal plane of a diffraction-limited optical telescope. Right: side projection of the photon distribution. The gaussian-shaped main distribution is surrounded by the Airy rings.*

This is the standard definition for visible light or any energy band where the telescope makes a direct detection of the photons, that is, photons coming from the source go through the whole optical system (including the Earth atmosphere) and finally generate a distribution on the focal plane. The PSF of an optical telescope generally has the shape that is illustrated in figure 1: most of the light follows a gaussian distribution but diffraction gives rise to several circles with decreasing intensity. This is the so-called Airy disk.

In the case of Cherenkov telescopes the detection is indirect. The actual photons coming from the source disappear in the atmosphere and their incident direction is reconstructed from the shower image on the telescope camera. Let us assume that each image gives rise to a single arrival direction. The PSF can be defined by analogy to the case of direct detection as the distribution of the reconstructed arrival directions.

2 Angular resolution

Point-like sources separated by an angle smaller than the angular resolution cannot be resolved. The Rayleigh criterion is the generally accepted criterion for the minimum resolvable detail. It is illustrated on figure 2. According to the Rayleigh criterion the angular resolution is the full width at half maximum of the PSF (FWHM).

Let us relate the angular resolution to some parameters that are widely used to describe the PSF. In a first approximation the PSF can be described with a bidimensional symmetric gaussian distribution.

$$\rho = K\ e^{-\frac{r^2}{2\sigma^2}} \tag{1}$$

where ρ is the event density and r is the angular distance to the source position.

A simple consequence of the definition is that the PSF falls in a factor $\sqrt{e}$ or 1.64 at an angular distance σ from the source. Hence the half width at half maximum (HWHM, that corresponds to a decrease in a factor 2 from the value at the center) is slightly larger than σ. In a first approximation σ=HWHM and **the angular resolution is roughly equal to 2σ.**

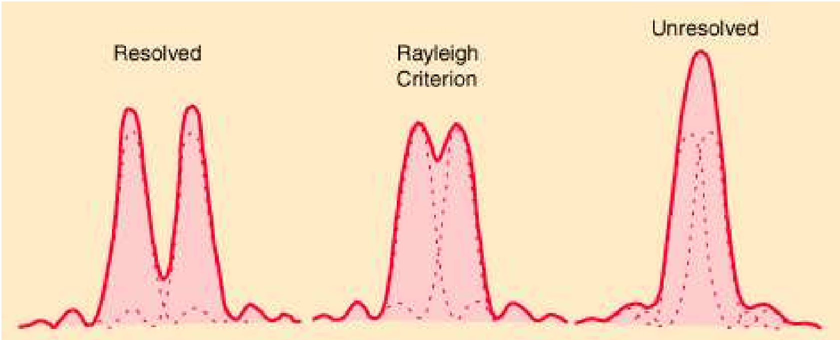

Figure 2: *Rayleigh criterion for the definition of angular resolution.*

The bidimensional gaussian can be integrated analytically. The volume under the function V (in our case the number of events) out to an angular distance R is given by:

$$V(<R) = -2\pi K\sigma^2\ [e^{-\frac{r^2}{2\sigma^2}}]_0^R \tag{2}$$

The volume under the whole function (R=∞) is equal to $2\pi K\sigma^2$. It follows that $\sim$40% of the events lie below 1σ and $\sim$85% of the events lie below 2σ. Note that these fractions are different than those for a unidimensional gaussian distribution (68% and 95% respectively). The volume below 1.5σ is roughly 70%: this value is sometimes used to characterize the PSF.

Let us consider now the projections onto the axes. If we project the bidimensional gaussian into any of the axes, we get another gaussian that results from the integration of the other variable. For example, the projection onto the Y-axis gives:

$$\rho(x) = \sqrt{2\pi}\cdot\sigma\cdot K\cdot e^{-\frac{x^2}{2\sigma^2}} \tag{3}$$

What is to say that the projection has a σ that is strictly equal to the σ of the bidimensional gaussian. Of course this σ does contain 68% of the events.

Cherenkov astronomers use sometimes the expression "RMS of the PSF". This expression is rather misleading, since strictly speaking one should refer to only one of the variables X or Y to define the RMS. It should be avoided or alternatively it should be mentioned explicitly that it is the RMS of any of the projections, which is in turn equal to σ.

In general Cherenkov astronomers do not use the Rayleigh criterion when they define the angular resolution and rather talk about σ. Sometimes they do not even mention that they are talking about σ. In order to make consistent comparisons with the performance of other telescopes, **I recommend to state always explicitly our definition whenever we talk about angular resolution, that is, use always a sentence like "angular resolution (σ of PSF)".**

3 Source position accuracy

When we are dealing with a **point-like source**, the error in the position of the source or **source position accuracy** is given not by σ but by the error in the mean, i.e. σ divided by the square root of the number of events. This is why the position of a stronger source is always better defined than that of a weak one, and why a longer observation results in an higher position accuracy.

It makes no sense to state the source position accuracy of a telescope without explicitly mentioning the flux of the source and the integration time of the observation, or equivalently the number of events.