A PROPOSED MODE OF OBSERVATION FOR MAGIC (REVISED VERSION)

W. Wittek
wittek@mppmu.mpg.de

18th May 2005

Abstract

Contents

1 Introduction

When a point source is observed by a telescope one is recording not only showers due to the gammas from the point source (source gammas) but also background showers due to the diffuse component of the cosmic ray flux (electrons, photons and hadrons). In order to separate this background from the source gammas one usually takes separately on and off data : for getting the on data the telescope is directed exactly to the source, for the off data the telescope is directed to a sky region which is sufficiently displaced from the source position and which hopefully does not contain strong gamma point sources. The source gammas are then essentially obtained by subtracting the off from the on data. With this procedure a considerable part of the observation time is used for taking off data.

One may gain observation time by trying to produce on and off data simultaneously. This is done in the so-called wobble mode : The telescope is directed not exactly to the source position but to a point which is displaced from it by an angle $\Delta\beta$. The sign of $\Delta\beta$ is changed periodically (say every 20 minutes, corresponding to 5° in right ascension) in order to collect off data not only from the region on one side of the source. The two wobble positions corresponding to the two signs of $\Delta\beta$ are called wobble position 1 and wobble position 2. $\Delta\beta$ should be chosen as a certain fraction of the field of view of the camera such that the source position is well inside the camera. The on data can now be obtained by analysing the shower images with respect to the point in the camera which corresponds to the source position, the off data by analysing the shower images with respect to some other point in the camera (this point will be called 'anti' source position in the following) which is not too close to the source position. As before, the source gammas are now essentially obtained by subtracting the off from the on data. The 'basic assumption' in this procedure is that the off data represent a good approximation of the background which is contained in the on data.

There are different ways of choosing the displacement $\Delta\beta$. $\Delta\beta$ may for example be taken as a fixed displacement

a) in the sky coordinates (δ, RA), or

b) in the local coordinates (Θ, φ), or

c) in the coordinates (x, y) in the camera system, where x is along the horizontal and y along the second axis of the camera plane.

A criterion for prefering one specific choice over another could be how well the above mentioned 'basic assumption' is fullfilled. How well it is fullfilled depends certainly on the choice of the 'anti' source position. For a given source position in the camera a reasonable choice of the 'anti' source position is the point in the camera which is obtained by reflecting the source position at the center of the camera. With respect to their position in the camera source and 'anti' source position are then completely symmetric. However, because the number and the properties of the showers depend strongly on the zenith angle Θ, the source and the 'anti' source positions are in general not symmetric with respect to the shower images. They are only symmetric if source and 'anti' source position correspond to the same zenith angle. The latter is only the case if the source position lies on the horizontal axis of the camera.

One could also define the 'anti' source position to be the point in the camera obtained by reflecting the source position at the y-axis of the camera. In this case source and 'anti' source position would have the same zenith angle, however source and 'anti' source positon would be closer together than with the previous choice of the 'anti' source position. In addition the distance between the two would in general be time dependent. Larger distances between the source and 'anti' source positions are desirable in order to reduce the contamination of the off data by source gammas.

From this discussion it appears that defining $\Delta\beta$ to be a fixed displacement along the horizontal axis (x-axis) in the camera is an optimal choice. This wobble mode is discussed in more detail in the subsequent section.

Remarks on wobble mode a) :
Here, when the source is tracked, the source positions moves along a circle around the camera center. The 'anti' source position (if defined as the point obtained by reflecting the source position at the center of the camera) has in general a different zenith angle than the source position. For a fixed sign of $\Delta\beta$, the sky region projected onto the camera does not change, although the sky image is rotating in the camera.

Remarks on wobble mode b) :
If $\Delta\beta$ is a fixed displacement in the zenith angle Θ the source will be on a fixed position on the y-axis of the camera. The 'anti' source position (if defined as the point obtained by reflecting the source position at the center of the camera) has always a different zenith angle than the source position. During tracking of the source, the sky image is rotating around the source position and the sky region projected onto the camera is changing continuously.

If $\Delta\beta$ is a fixed displacement along the direction of the azimuthal angle φ the source will in general be close to the x-axis of the camera, the distance from the x-axis depending on Θ. The 'anti' source position (if defined as the point obtained by reflecting the source position at the center of the camera) has always a different zenith angle than the source position, although the difference is in general small. During tracking of the source, the sky image is rotating and the sky region projected onto the camera is changing continuously.

2 Mode of observation proposed for MAGIC

2.1 Definition of the proposed wobble mode

Given an outer radius of the outer part of the MAGIC camera of 2°, a radius of the inner part of 1.25° and a radius of the trigger region of 0.8°, a value $\Delta\beta = \pm 0.4^\circ$ seems appropriate for the MAGIC telescope.

In the following the wobble mode is described which is proposed for MAGIC (see also [2]) :

- For zenith angles Θ of the source $\Theta > 0.4^\circ$ (see Fig. 1) :
  $\Delta\beta$ is a fixed displacement along the x-axis (horizontal axis) of the camera. Thus, by definition, the source is always at a fixed position on the x-axis of the camera. The 'anti' source position is defined as the point obtained by reflecting the source position at the center of the camera. It is thus also always at a fixed position on the x-axis of the camera. The two positions are interchanged when the sign of $\Delta\beta$ is reversed. The mode with positive $\Delta\beta$ is called wobble position 1, that with negative $\Delta\beta$ wobble position 2.

  With these definitions source and 'anti' source position have always the same zenith angle.

- For zenith angles Θ of the source $\Theta < 0.4^\circ$ (see Fig. 2) :
  In the limit $\Theta = 0$ any displacement is a displacement in Θ. Therefore the above definition of $\Delta\beta$ is not applicable for $\Theta < 0.4^\circ$. This can also be seen from eq. (37) below : there is no telescope orientation φ_0 for which $\tan\Theta < |x|$. It is natural to define $\Delta\beta$ in this case as a

fixed displacement along the y-axis in the camera, which corresponds to a fixed displacement in Θ (see eq. (40)). Thus, by definition, the source is always at a fixed position on the y-axis of the camera. The 'anti' source position, if defined as the point obtained by reflecting the source position at the center of the camera, is thus also always at a fixed position on the y-axis of the camera.

If (Θ, φ) are the current local coordinates of the source a negative $\Delta\beta$ implies an orientation (Θ_0, φ_0) of the telescope with $\Theta_0 = \Theta - \Delta\beta = \Theta + 0.4°$ and $\varphi_0 = \varphi$ (see eqs. (40) and (41)). The second wobble position, corresponding to a positive $\Delta\beta$, would give a telescope orientation with $\Theta_0 = \Theta - \Delta\beta = \Theta - 0.4° < 0$ and $\varphi_0 = \varphi$. In order to direct the telescope to this direction one either has to work in the so-called 'reverse' mode of the telescope (which allows negative values of Θ_0, see [1]) or one has to direct the telescope to the direction $\Theta_0 = -\Theta + \Delta\beta = -\Theta + 0.4° > 0$ and $\varphi_0 = \varphi + 180°$, which requires a rotation of the telescope by 180°. Because of the low $|\Theta_0|$ values involved neither of these solutions is very attractive. We therefore propose to use in the case of low Θ only one wobble position, the one corresponding to a negative $\Delta\beta$ (wobble position 3, see Fig. 2).

With these definitions the zenith angles of the source and 'anti' source position are always different (by $2 \cdot \Delta\beta = 0.8°$). However, because the number and the properties of the showers are essentially functions of $\cos\Theta$ and not of Θ, the difference in Θ at these small values of Θ is quite irrelevant.

The switch from low to high Θ need not be made exactly at $\Theta = |\Delta\beta| = 0.4°$ but also at some higher angle, say at 1°.

2.2 Properties of the proposed wobble mode

During tracking of the source, the sky image in the camera is rotating around the source position. The sky region projected onto the camera is therefore changing continuously and the 'anti' source position describes on the sky a circle around the source. In contrast to wobble mode a), one is collecting off data not only from a fixed sky region near the source but from a more extended sky region, with constant distance from the source.

For practical purposes and for defining the telescope guidance one has to know which displacements in sky coordinates (δ, RA) and in local coordinates (Θ, φ) are implied by a fixed displacement in the coordinates (x, y) of the camera system. This question is discussed in the subsequent sections.

3 Coordinate systems and transformations

3.1 Coordinate systems

Three coordinate systems will be considered :

- System A : A local coordinate system (x_A, y_A, z_A) in which the local (zenith and azimuthal) angles (Θ, φ) are defined
- System B : An equatorial coordinate system (x_B, y_B, z_B) in which the sky coordinates (δ, ϕ), i.e declination and hour angle, are defined. ϕ is defined to be zero when the source is culminating at the geographical longitude of the telescope. The relation between the hour angle ϕ and the right ascension RA is

$$\phi = -RA + c_0 + c_1 \cdot t \tag{1}$$

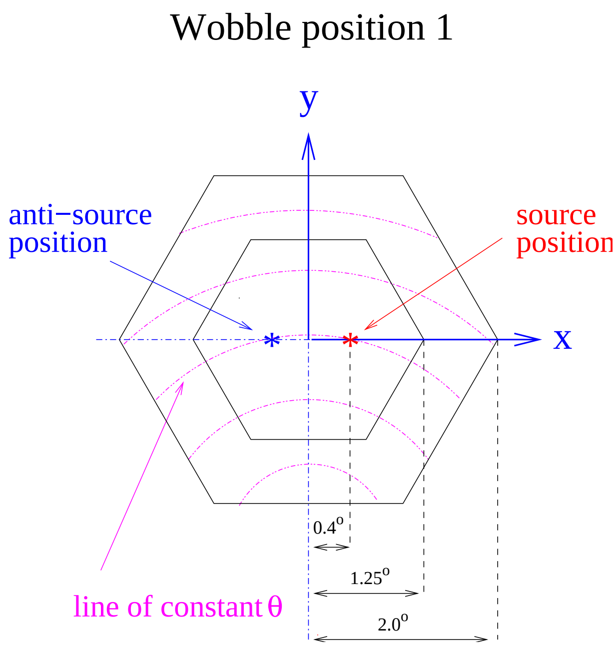

Figure 1: Source and 'anti' source positions in the camera system for wobble position 1

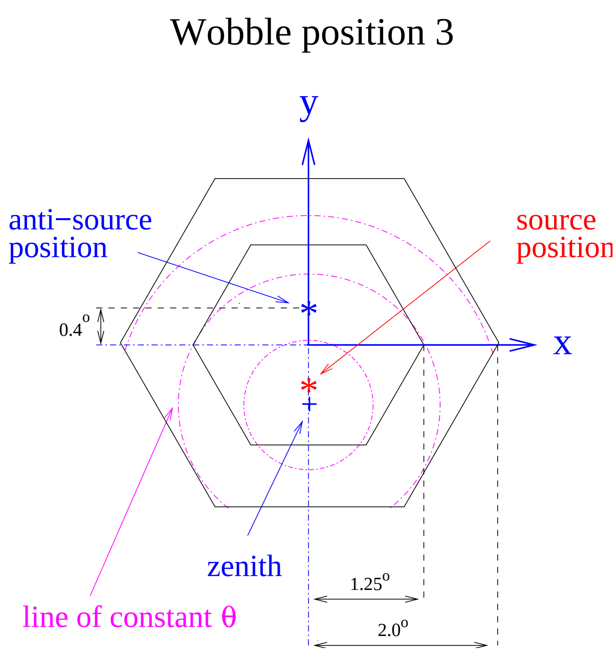

Figure 2: Source and 'anti' source positions in the camera system for wobble position 3

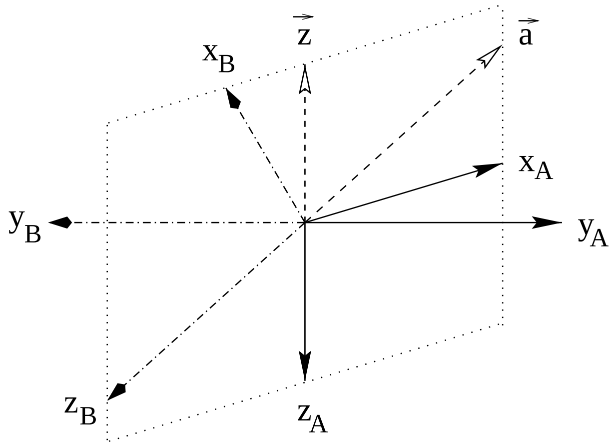

Figure 3: Directions of the axes (x_A, y_A, z_A) and (x_B, y_B, z_B) of the systems A and B respectively. The plane formed by the earth-rotation axis $\vec{a}$ and the zenith direction $\vec{z}$ contains also the directions x_A, z_A and x_B, z_B. The directions y_A and y_B are perpendicular to this plane. The zenith direction $\vec{z}$ is opposite to z_A, and $\vec{a}$ is opposite to z_B.

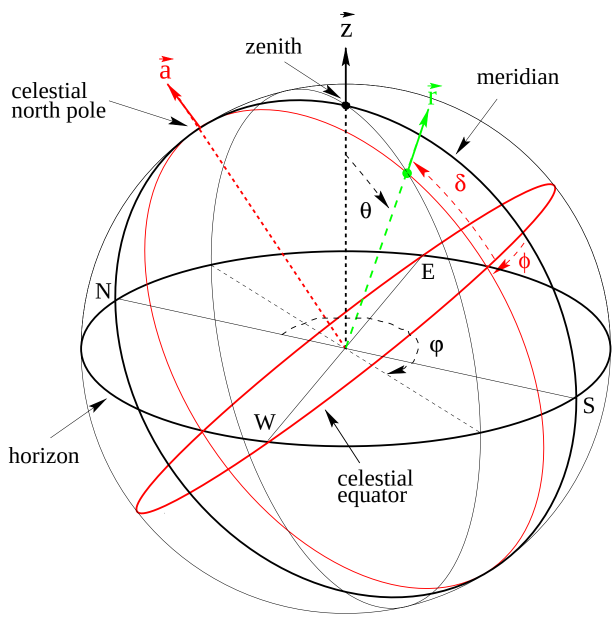

Figure 4: Definition of the zenith angle Θ, the azimuthal angle φ, the declination δ and the hour angle ϕ. The vectors $\vec{a}, \vec{z}$ and $\vec{r}$ denote the earth-rotation axis (pointing to the celestial north pole), the zenith direction and the position of a point on the sky respectively.

where c_0 andc_1 are independent of the source and independent of the time t with $c_1 = \dfrac{360^\circ}{0.9972696 \times 24\ \mathrm{h}}$. If RA and ϕ are known for an arbitrary point on the sky at the time t the constant c_0 can be determined and the relation between RA and ϕ is known for all sky directions, at all times.

- System T : A telescope coordinate system (x_T, y_T, z_T)

The definition of the systems A and B is given in [1] and it can also be seen from Figs. 3 and 4. The vectors $\vec{a}$, $\vec{z}$ and $\vec{r}$ denote the earth-rotation axis (pointing to the celestial north pole), the zenith direction and the position of a point on the sky respectively.

The telescope system is defined as :

- x-axis in the direction of $\vec{e}_x = \dfrac{\vec{r}_0 \times \vec{z}}{|\vec{r}_0 \times \vec{z}|}$
- y-axis in the direction of $\vec{e}_y = \vec{e}_x \times \vec{r}_0$
- z-axis in the direction of $\vec{e}_z = -\vec{r}_0$

where $\vec{r}_0$ represents the direction the telescope is pointing to.

For understanding the definition of the telescope system the following consideration may be useful : When the observer is looking from the center of the reflector in the direction of the telescope axis (towards the camera) the x_T-axis is pointing horizontally to the right, the y_T-axis upwards and the z_T-axis towards the observer.

The origin of all three systems is assumed to be in the center of the reflector.

A natural definition of the camera plane is the plane perpendicular to $\vec{e}_z$ at $z_T = -R_C$, where R_C is the distance of the camera from the reflector center. For convenience, a fictive camera system (x_C, y_C) is defined in the plane at $z_T = -1$ with its center on the $\vec{e}_z$ axis. With this definition small x_C and small y_C near $(x_C, y_C) = (0, 0)$ in the fictive camera system correspond directly to angles in the sky (see eqs. (35) and (38)) :

$$\Delta x_C = -\sin\Theta_0 \cdot \Delta \tan\varphi \simeq -\sin\Theta_0 \cdot \Delta\varphi \tag{2}$$

$$\Delta y_C = \Delta \tan\Theta \simeq \Delta\Theta \tag{3}$$

The position (x_{trueC}, y_{trueC}) in the true camera plane is obtained from (x_C, y_C) by

$$x_{trueC} = R_C \cdot x_C \qquad y_{trueC} = R_C \cdot y_C \tag{4}$$

3.2 Transformations between different systems

The reader is also refered to the TDAS note 05-07, which contains a more comprehensive discussion of the various transformations.

a) Transformation from local coordinates (Θ, φ) to coordinates (x_C, y_C) in the fictive camera system

The representations of the vectors $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$, $\vec{r}_0$ and of some sky direction $\vec{r}^{\,orig}$ in system A are

$$\vec{e}_{x,A} = \begin{pmatrix} -\sin\varphi_0 \\ \cos\varphi_0 \\ 0 \end{pmatrix} \qquad \vec{e}_{y,A} = \begin{pmatrix} -\cos\Theta_0 \cdot \cos\varphi_0 \\ -\cos\Theta_0 \cdot \sin\varphi_0 \\ -\sin\Theta_0 \end{pmatrix} \tag{5}$$

$$\vec{r}_{0,A} = \begin{pmatrix} \sin\Theta_0 \cdot \cos\varphi_0 \\ \sin\Theta_0 \cdot \sin\varphi_0 \\ -\cos\Theta_0 \end{pmatrix} = -\vec{e}_{z,A} \qquad \vec{r}_A^{\,orig} = \begin{pmatrix} \sin\Theta \cdot \cos\varphi \\ \sin\Theta \cdot \sin\varphi \\ -\cos\Theta \end{pmatrix} = \begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix} \tag{6}$$

It follows that the components of $\vec{r}^{\,orig}$ in the telescope system are

$$\vec{r}_T^{\,orig} = \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} = \begin{pmatrix} \vec{e}_{x,A} \cdot \vec{r}_A^{\,orig} \\ \vec{e}_{y,A} \cdot \vec{r}_A^{\,orig} \\ \vec{e}_{z,A} \cdot \vec{r}_A^{\,orig} \end{pmatrix} = M \cdot \vec{r}_A^{\,orig} \tag{7}$$

with

$$M = \begin{pmatrix} -\sin\varphi_0 & \cos\varphi_0 & 0 \\ -\cos\Theta_0 \cos\varphi_0 & -\cos\Theta_0 \sin\varphi_0 & -\sin\Theta_0 \\ -\sin\Theta_0 \cos\varphi_0 & -\sin\Theta_0 \sin\varphi_0 & \cos\Theta_0 \end{pmatrix} \tag{8}$$

Thus

$$\vec{r}_T^{\,orig} = \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} = \begin{pmatrix} \sin\Theta \cdot \sin(\varphi - \varphi_0) \\ \sin\Theta_0 \cos\Theta - \cos\Theta_0 \cdot \sin\Theta \cdot \cos(\varphi - \varphi_0) \\ -\cos\Theta_0 \cos\Theta - \sin\Theta_0 \cdot \sin\Theta \cdot \cos(\varphi - \varphi_0) \end{pmatrix} \tag{9}$$

Note that these are the components of the original direction $\vec{r}^{\,orig}$ in the telescope system (before the reflection at the reflector). The reflection at the reflector is equivalent to a rotation of $\vec{r}^{\,orig}$ by 180° around the telescope orientation $\vec{r}_0$:

$$\vec{r}_T^{\,ref} = \begin{pmatrix} x^{ref} \\ y^{ref} \\ z^{ref} \end{pmatrix} = \begin{pmatrix} -x_T \\ -y_T \\ z_T \end{pmatrix} = S \cdot \vec{r}_T^{\,orig} \tag{10}$$

with

$$S = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{pmatrix} = S^{-1} \tag{11}$$

Using (7) the components of $\vec{r}^{\,ref}$ in the telescope system (after reflection) can be written as

$$\vec{r}_T^{\,ref} = \begin{pmatrix} x^{ref} \\ y^{ref} \\ z^{ref} \end{pmatrix} = S \cdot \vec{r}_T^{\,orig} = S \cdot M \cdot \vec{r}_A^{\,orig} \tag{12}$$

$$= \begin{pmatrix} -\sin\Theta \cdot \sin(\varphi - \varphi_0) \\ -\sin\Theta_0 \cos\Theta + \cos\Theta_0 \cdot \sin\Theta \cdot \cos(\varphi - \varphi_0) \\ -\cos\Theta_0 \cos\Theta - \sin\Theta_0 \cdot \sin\Theta \cdot \cos(\varphi - \varphi_0) \end{pmatrix} \tag{13}$$

$$= \begin{pmatrix} -\sin\Theta \cdot \sin(\varphi - \varphi_0) \\ \sin(\Theta - \Theta_0) - \cos\Theta_0 \cdot \sin\Theta \cdot [1 - \cos(\varphi - \varphi_0)] \\ -\cos(\Theta - \Theta_0) + \sin\Theta_0 \cdot \sin\Theta \cdot [1 - \cos(\varphi - \varphi_0)] \end{pmatrix} \tag{14}$$

The coordinates (x_C, y_C) in the fictive camera system are now obtained by determining the intersection point $\vec{r}_T^{\,cam}$ of the line $(\tau \cdot \vec{r}_T^{\,ref})$ with the plane $z_T = -1$:

$$\vec{r}_T^{\,cam} = \begin{pmatrix} x_C \\ y_C \\ -1 \end{pmatrix} = \frac{-1}{z^{ref}} \cdot \begin{pmatrix} x^{ref} \\ y^{ref} \\ z^{ref} \end{pmatrix} \tag{15}$$

or

$$\begin{pmatrix} x_C \\ y_C \end{pmatrix} = \frac{1}{\sqrt{1 - (x^{ref})^2 - (y^{ref})^2}} \cdot \begin{pmatrix} x^{ref} \\ y^{ref} \end{pmatrix} \tag{16}$$

since $\vec{r}_T^{\,ref}$ is a unit vector. Note that the scaling factor $(-1/z^{ref})$ is close to $+1$, because (due to the small field of view of the camera) the directions (Θ, φ) and (Θ_0, φ_0) are close.

b) Transformation from (x_C, y_C) to (Θ, φ)

The point (x_C, y_C) in the fictive camera system is the point $(x_C, y_C, -1)$ in the telescope system, from which the unit vector $\vec{r}_T^{\,ref}$ is derived by

$$\vec{r}_T^{\,ref} = \begin{pmatrix} x^{ref} \\ y^{ref} \\ z^{ref} \end{pmatrix} = \frac{1}{\sqrt{1 + x_C^2 + y_C^2}} \cdot \begin{pmatrix} x_C \\ y_C \\ -1 \end{pmatrix} \tag{17}$$

Inverting (12) the original direction $\vec{r}_A^{\,orig}$ in system A is given by

$$\vec{r}_A^{\,orig} = \begin{pmatrix} \sin\Theta \cdot \cos\varphi \\ \sin\Theta \cdot \sin\varphi \\ -\cos\Theta \end{pmatrix} = M^{-1} \cdot S^{-1} \cdot \vec{r}_T^{\,ref} = M^T \cdot S \cdot \vec{r}_T^{\,ref} \tag{18}$$

$$= \begin{pmatrix} \sin\varphi_0 & \cos\Theta_0 \cdot \cos\varphi_0 & -\sin\Theta_0 \cdot \cos\varphi_0 \\ -\cos\varphi_0 & \cos\Theta_0 \cdot \sin\varphi_0 & -\sin\Theta_0 \cdot \sin\varphi_0 \\ 0 & \sin\Theta_0 & \cos\Theta_0 \end{pmatrix} \begin{pmatrix} x^{ref} \\ y^{ref} \\ z^{ref} \end{pmatrix} \tag{19}$$

c) Calculation of the telescope orientation (Θ_0, φ_0) **from** (x_C, y_C) **and** (Θ, φ)

In a) and b) transformations for a fixed orientation (Θ_0, φ_0) (or $\vec{r}_{0,A}$) of the telescope were considered, namely between a direction (Θ, φ) (or $\vec{r}_A^{\,orig}$) and a position (x_C, y_C) in the fictive camera system. Alternatively one may want to determine the telescope orientation $\vec{r}_{0,A}$ from the coordinates (x_C, y_C) and (Θ, φ) of a source at a given time.

Using (17) and (10) $\vec{r}_T^{\,orig}$ can be calculated from (x_C, y_C) as

$$\vec{r}_T^{\,orig} = \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} = S^{-1} \cdot \vec{r}_T^{\,ref} = \frac{1}{\sqrt{1 + x_C^2 + y_C^2}} \cdot \begin{pmatrix} -x_C \\ -y_C \\ -1 \end{pmatrix} \tag{20}$$

The basic equation needed for computing (Θ_0, φ_0) (or $\vec{r}_{0,A}$) is equation (7)

$$\vec{r}_T^{\,orig} = \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} = M \cdot \vec{r}_A^{\,orig} \tag{21}$$

which implies

$$\vec{r}_A^{\,orig} = \begin{pmatrix} \sin\Theta \cdot \cos\varphi \\ \sin\Theta \cdot \sin\varphi \\ -\cos\Theta \end{pmatrix} = \begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix} = M^{-1} \cdot \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} = M^T \cdot \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} \tag{22}$$

In (21) $\vec{r}_T^{\,orig}$ and $\vec{r}_A^{\,orig}$ are known, and M contains the quantities (Θ_0, φ_0) to be determined (see eq.(8)). (21) or (22) are systems of 3 equations with 2 unknowns. Because $\vec{r}_A^{\,orig}$ and $\vec{r}_T^{\,orig}$ are unit vectors only 2 of the 3 equations are independent. M can also be written as

$$M = V \cdot H \tag{23}$$

with

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Theta_0 & -\sin\Theta_0 \\ 0 & \sin\Theta_0 & \cos\Theta_0 \end{pmatrix} \qquad H = \begin{pmatrix} -\sin\varphi_0 & \cos\varphi_0 & 0 \\ -\cos\varphi_0 & -\sin\varphi_0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{24}$$

From (22) one finds

$$\pm\sqrt{x_A^2 + y_A^2 - x_T^2} = \cos\Theta_0 \cdot y_T + \sin\Theta_0 \cdot z_T \tag{25}$$

$$z_A = -\sin\Theta_0 \cdot y_T + \cos\Theta_0 \cdot z_T \tag{26}$$

or

$$\sin\Theta_0 = \frac{-y_T \cdot z_A \pm z_T \sqrt{x_A^2 + y_A^2 - x_T^2}}{y_T^2 + z_T^2} \tag{27}$$

$$\cos\Theta_0 = \frac{+z_T \cdot z_A \pm y_T \sqrt{x_A^2 + y_A^2 - x_T^2}}{y_T^2 + z_T^2} \tag{28}$$

The sign in front of the square root has to be chosen such that Θ_0 lies between 0 and π. Relation (28), which follows directly from (26), is actually sufficient for determining Θ_0.
Knowing Θ_0 the matrix V and the vector $(V^{-1} \cdot \vec{r}_T^{\,orig})$ can be calculated :

$$V^{-1} \cdot \vec{r}_T^{\,orig} = \begin{pmatrix} x_S \\ y_S \\ z_S \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Theta_0 & \sin\Theta_0 \\ 0 & -\sin\Theta_0 & \cos\Theta_0 \end{pmatrix} \cdot \begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} \tag{29}$$

$$= \begin{pmatrix} x_T \\ \cos\Theta_0 \cdot y_T + \sin\Theta_0 \cdot z_T \\ -\sin\Theta_0 \cdot y_T + \cos\Theta_0 \cdot z_T \end{pmatrix} \tag{30}$$

According to (21), (23) and (24) this vector is equal to

$$V^{-1} \cdot \vec{r}_T^{\,orig} = \begin{pmatrix} x_S \\ y_S \\ z_S \end{pmatrix} = H \cdot \vec{r}_A^{\,orig} = \begin{pmatrix} -\sin\varphi_0 & \cos\varphi_0 & 0 \\ -\cos\varphi_0 & -\sin\varphi_0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix} \tag{31}$$

Solving (31) for $\sin\varphi_0$ and $\cos\varphi_0$ yields

$$\sin\varphi_0 = \frac{-x_S \cdot x_A - y_S \cdot y_A}{x_A^2 + y_A^2} \tag{32}$$

$$\cos\varphi_0 = \frac{+x_S \cdot y_A - y_S \cdot x_A}{x_A^2 + y_A^2} \tag{33}$$

The telescope orientation is completely given by the equations (27), (28), (32) and (33).

d) Special transformations

If a point (x_C, y_C) in the fictive camera is given, equations (14) and (15) may be used

- to determine the original source direction (Θ, φ) for a fixed telescope orientation (Θ_0, φ_0), or
- to calculate the telescope direction (Θ_0, φ_0) for a fixed source direction (Θ, φ)

From (14) and (15) one finds that for a given telescope orientation (Θ_0, φ_0) a source position $(x_C, y_C) = (x_C, 0)$ on the x-axis of the fictive camera system corresponds to an original source direction (Θ, φ), where Θ and φ are given by

$$\cos\Theta = \frac{\cos\Theta_0}{\sqrt{1 + x_C^2}} \tag{34}$$

$$\tan(\varphi - \varphi_0) = \frac{-x_C}{\sin\Theta_0} \tag{35}$$

By inverting this equation one obtains for a given source direction (Θ, φ) with the position $(x_C, 0)$ in the fictive camera system the telescope orientation (Θ_0, φ_0) by

$$\cos\Theta_0 = \cos\Theta \cdot \sqrt{1+x_C^2} \tag{36}$$

$$\tan(\varphi-\varphi_0) = \frac{-x_C}{\sqrt{\sin^2\Theta - x_C^2\cos^2\Theta}} \tag{37}$$

For a given telescope orientation (Θ_0, φ_0), a source position $(x_C, y_C) = (0, y_C)$ on the y-axis of the fictive camera system corresponds to a direction (Θ, φ), where Θ and φ are given by

$$\tan(\Theta-\Theta_0) = y_C \tag{38}$$

$$\varphi = \varphi_0 \tag{39}$$

By inverting this equation one obtains for a given source direction (Θ, φ) with the position $(0, y_C)$ in the fictive camera system the telescope orientation (Θ_0, φ_0) by

$$\tan(\Theta-\Theta_0) = y_C \tag{40}$$

$$\varphi_0 = \varphi \tag{41}$$

4 Tracking of a source

4.1 Calculation of the telescope orientation

For a given source at the position (δ, RA) and for a given absolute time t the guiding system will calculate the sky coordinates (δ, ϕ) (or $\vec{r}_B$) and the current local coordinates (Θ, φ) (or $\vec{r}_A$) of the source, including all corrections (precession, nutation, refraction, etc.). Depending on the wobble position (1, 2 or 3), which fixes (x_C, y_C), the telescope orientation (Θ_0, φ_0) (or $\vec{r}_{0,A}$) can be calculated using eqs. (36), (37) and (40), (41) respectively. With this telescope orientation the source should now appear at the desired position in the camera.

It may be of interest to know the sky position (δ_0, ϕ_0) (or $\vec{r}_{0,B}$) corresponding to the telescope orientation (Θ_0, φ_0) (or $\vec{r}_{0,A}$). Knowing the exact sky coordinates $\vec{r}_B$ of the source, the (precisely) calculated local coordinates $\vec{r}_A$ of the source and the (precisely) calculated local coordinates $\vec{r}_{0,A}$ of the telescope orientation one can calculate the approximate sky coordinates (δ_0, ϕ_0) (or $\vec{r}_{0,B}$) of the telescope orientation :

Using the transformation matrix A for transformations from system A to system B and vice versa (see eq. (3) from [1]) $\vec{r}_{0,B}$ can be written as

$$\vec{r}_{0,B} \simeq \vec{r}_B + A\cdot(\vec{r}_{0,A} - \vec{r}_A) \tag{42}$$

or

$$\begin{pmatrix} \cos\delta_0\cos\phi_0 \\ \cos\delta_0\sin\phi_0 \\ -\sin\delta_0 \end{pmatrix} \simeq \begin{pmatrix} \cos\delta\cos\phi \\ \cos\delta\sin\phi \\ -\sin\delta \end{pmatrix} + \begin{pmatrix} a_3 & 0 & -a_1 \\ 0 & -1 & 0 \\ -a_1 & 0 & -a_3 \end{pmatrix} \begin{pmatrix} \sin\Theta_0\cos\varphi_0 - \sin\Theta\cos\varphi \\ \sin\Theta_0\sin\varphi_0 - \sin\Theta\sin\varphi \\ -\cos\Theta_0 \quad +\cos\Theta \end{pmatrix} \tag{43}$$

Here $a_1 = \cos(Lat)$ and $a_3 = -\sin(LaT)$, where Lat is the geographical latitude of the observing telescope. For La Palma $Lat = 28.76189°$ giving $a_1 = 0.876627$ and $a_3 = -0.481171$.

Equation (43) yields only an approximate telescope orientation (δ_0, ϕ_0) because the corrections mentioned above are not taken into account. However, since only the difference of two (close) directions is transformed the corrections will cancel to a large extent.

The right ascension RA_0 corresponding to the sky coordinate ϕ_0 at the time t is obtained by

$$RA_0 \;=\; RA \;+\; (\phi - \phi_0) \tag{44}$$

where RA and ϕ are the exact sky coordinates of the source at the time t.

For calculating (δ_0, ϕ_0) according to (43), or RA_0 according to (44), in particular ϕ has to be known. If ϕ is not provided by the guiding system it can be calculated from RA using (1).

4.2 Checking the telescope orientation

By means of some monitoring system (see [3]) it may be possible to measure the actual telescope orientation $\overline{\vec{r}_0}$ at some time t. If $\overline{\vec{r}_0}$ differs too much from the desired direction $\vec{r}_0$ the telescope orientation (or the tracking speed) has to be readjusted. $\overline{\vec{r}_0}$ may be given in terms of local coordinates $(\overline{\Theta_0}, \overline{\varphi_0})$ (or $\overline{\vec{r}_{0,A}}$) or in terms of sky coordinates $(\overline{\delta_0}, \overline{RA_0})$. $\overline{\vec{r}_0}$ may also be infered from the current cartesian coordinates (x_C, y_C) of a star in the CCD camera.

For readjusting the telescope orientation $\overline{\vec{r}_0}$ and $\vec{r}_0$ should obviously be represented in local coordinates (system A). If they are given in a different representation they have to be transformed into local coordinates :

a) The monitoring system provides $(\overline{\delta_0}, \overline{RA_0})$:
At a given time t the local coordinates of the actual telescope orientation $\overline{\vec{r}_{0,A}}$ corresponding to $(\overline{\delta_0}, \overline{RA_0})$ can be calculated in the usual way, including all corrections.

b) The monitoring system provides (x_C, y_C) of a star :
Knowing the sky coordinates (δ, RA) of the star and the time t at which (x_C, y_C) was measured the local coordinates (Θ, φ) (or $\vec{r}_A^{\,orig}$) of the star are calculated in the usual way, including all corrections. The actual telescope orientation $\overline{\vec{r}_{0,A}}$ is computed from (x_C, y_C) and $\vec{r}_A^{\,orig}$ as described in section 3.2c).

After proper calibration of the shaft encoders (see [3]), the shaft encoder values may be used to check the telescope orientation. As demonstrated in b), the CCD camera (see [3]) provides another way of checking the telescope orientation.

5 Consequences for the trigger

All trigger conditions which are rotational symmetric with respect to the camera center are not affected by the wobble mode. These are for example the trigger conditions which are based on the shapes of the shower images.

Attention must be paid to the trigger conditions which depend on the source position in the camera, i.e those which use the orientation of the shower image relative to the source position or the distance of the shower image from the source position. One has to make sure that these conditions do not destroy the symmetry between the source and the 'anti' source position, which is essential for the determination of the background in the source region. Symmetry can be achieved if the trigger conditions are symmetrized with respect to source and 'anti' source position.

References

[1] W. Wittek, MAGIC-TDAS 00-11 (2000)

[2] R. Böck et al., MAGIC-TDAS 01-03 (2001)

[3] R. Böck et al., MAGIC-TDAS 01-09 (2001)