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Conversion Factor Calibration for MAGIC Based on the Use of Measured F-Factors of PMTs

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Abstract

For the absolute sensitivity calibration of MAGIC it is necessary to know the mirror reflectivity, the quantum efficiency of the PMTs, the photoelectron collection efficiency on the 1st dynode as well as the conversion factor from a photoelectron (*phe*) released from the photocathode of a given PMT to digital counts of the charge/amplitude measuring system. Below I outline the possible ways for the latter calibration.

1 Introduction

To compare measured by the telescope data with the Monte Carlo simulations one needs to know the overall conversion factor from Cherenkov photons, hitting the reflector of the telescope, to the digital counts of the data acquisition system. The conversion factor can be measured in several ways, as for example, by measuring the response of the telescope to μ-induced Cherenkov ring images (see Rose, 1995) or by measuring the response of the signal measuring chain in the imaging camera to a distant source of light flashes of calibrated intensity (see Frass, et al., 1997). Both mentioned calibrations are incomplete because as a result one obtains just a number which relates *the mean number of incident photons* to the *mean value of measured charge*, measured in relative units of ADC channels. This alone is not enough in order compare *the shape of a measured distribution with that of the simulated distribution* (with all their fluctuations). Only after normalizing to the total gain of the signal channels one can convert the measured charge into the number of *phe*s impinging onto the first dynode of PMTs. Note that it is the number of *phe*s which fluctuates (and this will be reflected in the fluctuations of the digital counts) and it is a real absolute measurable. Statistical fluctuations of charge at the anode of a PMT has two main sources. The first one is due to the fluctuation of the number of *phe*s released from the photocathode (this is a binomial process) and the second one is due to the fluctuations in the charge multiplication process in the dynode structure. Clear understanding of both processes is decisive for correct interpretation of the measured data. Measurement of the number of *phe*s will allow one to disentangle the changes in the conversion factor due to the changes in the mirror reflectivity and the photon detection efficiency (light guide efficiency plus quantum efficiency plus the *phe* detection efficiency on the first dynode) from that of due to the variation of gain in the signal amplification chain. In the absence of the measurement of the *phe* to digital count conversion factor one can not follow the changes in the PMT gain due to, for example, short/long-term drifts or ageing. Due to the latter effects, for example, the PMTs will produce varying charge at the output for the same incident flux of photons. As a result one will mistakenly ascribe the change in the output measured charge to the lower/higher photon flux falling onto the reflector. As a conclusion it is clear that a simple regular procedure shell be developed and applied to the daily measured data of the telescope for following the variations of the *phe* to digital counts conversion factor providing thus good calibration accuracy. The *phe* to digital count conversion factor can be measured directly by means of providing additional amplifiers (gain) in the signal channel in order to amplify and resolve the single *phe* amplitude distribution or with the aid of measuring the excess-noise factor F of PMTs. I will concentrate myself on the latter option while the direct measuring method is illuminated in the article (Mirzoyan, Fernandez, Lorenz, et al., 1995).

2 The Amplitude Resolution of a PMT

The amplitude resolution of PMTs were studied already in the 50's in connection with the amplitude resolution of scintillation spectrometers. Below I will show a compilation of the main results of these studies following the book in ref. (Birks, 1964) while the most comprehensive study seems to be that of (Breitenberger, 1955).

2.1 Theoretical Studies

Let us denote the mean value of a quantity X by $\overline{X}$, its variance by $Var(X)$, where

$$Var(X) = \overline{X^2} - \overline{X}^2 \tag{1}$$

($Var(X)$ also is known as *squared standard deviation*) and its fractional variance by $v(X)$, where

$$v(X) = \frac{Var(X)}{\overline{X}^2} \tag{2}$$

Note that for the Poisson distribution one has

$$Var(X) = \overline{X} \tag{3}$$

and

$$v(X) = \frac{1}{\overline{X}} \tag{4}$$

Consider now a cascade of X_1, X_2, $\cdots$, X_n events where X_1 happens first and gives a score δ_{X_1}, then X_2 happens exactly δ_{X_1} times and gives a score δ_{X_2} and so forth, X_n happens only $\delta_{X_{n-1}}$ times and gives a score δ_{X_n} and the scores of all X_1 trials are added up to give the cascade score δ_X. For such a cascade of n events, by using the probability generating functions it can be shown (Shockley and Pierce, 1938) that

$$\overline{\delta}_X = \overline{\delta}_{X_1} \cdot \overline{\delta}_{X_2} \cdot \overline{\delta}_{X_3} \cdots \overline{\delta}_{X_n} \tag{5}$$

and

$$v(X) = v(X_1) + \frac{v(X_2)}{\overline{\delta}_{X_1}} + \cdots + \frac{v(X_n)}{\overline{\delta}_{X_1} \cdot \overline{\delta}_{X_2} \cdots \overline{\delta}_{X_{n-1}}} \tag{6}$$

2.2 Electron Multiplication by the System of Dynodes

Now let us apply the above obtained results to the process of a single *phe* (released by the photocathode of a PMT) multiplication in a PMT with n dynodes. Let g (with the corresponding sub-index) be the secondary electron amplification factor of a given dynode. The average gain $\overline{G}$ of the PMT, according to equation (5), can be written as:

$$\overline{G} = \overline{g_1} \cdot \overline{g_2} \cdots \overline{g_n} \tag{7}$$

and its fractional variance, according to equation (6), as:

$$v(G) = v(g_1) + \frac{v(g_2)}{\overline{g_1}} + \cdots + \frac{v(g_n)}{\overline{g_1} \cdot \overline{g_2} \cdots \overline{g_{n-1}}} \tag{8}$$

One can see that the first dynode has a dominant contribution into the total fractional variance $v(G)$ and all the other dynodes contribute much less (usually in order to provide peaked single *phe* distribution at the output one selects PMTs for which the gain of the first dynode $\overline{g_1}$ at the specified voltage is ≥ 6, so already the second term in (8) is 6 times less than the first one if the fractional variances are comparable). This should be one of the reasons why the PMT manufacturers measure the gain g_1 of the first dynode and show its value on the PMT data tickets. If we assume that all the PMT dynodes operate at the same mean gain $\overline{g}$ and have the same variance $v(g)$ then one can simplify the equation (8):

$$v(G) = v(g) \cdot \frac{\overline{g}^n - 1}{(\overline{g} - 1) \cdot \overline{g}^{n-1}} \tag{9}$$

Because of $\overline{g}^n \gg 1$, one can further simplify the equation (9) and obtain

$$v(G) = v(g) \cdot \frac{\overline{g}}{\overline{g} - 1} \tag{10}$$

Note that $v(G)$ in equation (10) is independent on the number of dynodes n. One can see from equation (10) that the total fractional variance of the PMT gain (under the assumption of identical dynodes and potential differencies) is essentially equal to the fractional variance of a single dynode; the rest of the dynode system enhances the fractional variance by $\frac{\overline{g}}{\overline{g}-1}$ times which, for example, is equal to 1.25 for a dynode gain of 5. If we assume now, as a *Gedanken experiment*, that the dynode gain has a Poisson variance (which is not true, in reality the gain has always larger than Poisson variance), i.e. $v(g) = \frac{1}{g}$ then

$$v(G) = (\frac{\sigma_G^2}{\overline{G}^2}) = \frac{1}{\overline{g} - 1} \tag{11}$$

For the above mentioned assumptions and, for example, for the $\overline{g} = 5$ one obtains a resolution of

$$\frac{\sigma_G}{\overline{G}} = 0.5 \tag{12}$$

Usually one operates the PMT under higher photocathode to 1st dynode potential compared to the rest of the interdynode potentials (this improves the focusing of *phe*s onto the first dynode as well as increases the number of secondary *phe*s hit out from the latter). Therefore one can simplify the equation (8) to the form

$$v(G) = v(g_1) + v(g) \cdot \frac{\overline{g}}{\overline{g_1} \cdot (\overline{g} - 1)} \tag{13}$$

And again, under the (non-realistic) assumption of Poisson variance of the gain one can come to the following simplification

$$v(G) = \frac{\overline{g}}{\overline{g_1} \cdot (\overline{g} - 1)} \tag{14}$$

Let us assume, for example, a gain of 8 for the first dynode and a gain of 5 for the rest of the dynodes (a realistic scenario). One can calculate then a resolution of

$$\frac{\sigma_G}{\overline{G}} = 0.395 \approx 0.40 \tag{15}$$

Note that the calculated values in (12) and in (15) can be seen as the resolutions of the corresponding single *phe* distributions (under the assumption of Poisson variance of the gain and therefore they can be seen as lower limits in the resolutions).

2.3 Resolution of a PMT for Monochromatic Light

Assume a bunch of N optical photons of a given wavelength λ_0 from a thermal source impinging on the PMT photocathode. A fraction η of these photons will be converted into *phe*s (η is the usual quantum efficiency) and subsequently collected on the surface of the first dynode with the efficiency β (β is the *phe* collection efficiency of the first dynode). The product $\eta \cdot \beta = p$ is called *photon detection efficiency*. These detected *phe*s are G times multiplied in the PMT dynode system and produce at the output (anode) a charge Q. One can write:

$$Q = N \cdot p \cdot G \tag{16}$$

which, similar to equations (5) and (6), can be considered as a cascade event. All three parameters in the right side of equation (16) are subject to statistical variations and thus they produce variations in the value of Q. Let us *assume for simplicity that the photon detection efficiency p in equation (16) does not depend to where on the photocathode surface area and under which angle a phe is knocked out.* Under these assumptions one can write:

$$\overline{Q} = \overline{N} \cdot p \cdot \overline{G} \tag{17}$$

(By using the probability generating function for p it can be shown that the average score is p and its variance is $p \cdot (1-p)$). By applying the formalism of the cascade events to (16) one can obtain:

$$v(Q) = v(N) + \frac{p \cdot (1-p)}{\overline{N} \cdot p^2} + \frac{v(G)}{\overline{N} \cdot p} \tag{18}$$

and after re-grouping the terms one can write

$$v(Q) = \left(v(N) - \frac{1}{\overline{N}}\right) + \frac{1}{\overline{N} \cdot p} \cdot (1 + v(G)) \tag{19}$$

If the light source has a Poisson variance (this is true for thermal sources of light; also it is true for the scintillation light and the LED light) then the first bracketed term in the above equation vanishes and one obtains a simplification of

$$v(Q) = \frac{1}{\overline{N} \cdot p} \cdot (1 + v(G)) \tag{20}$$

The product $\overline{N} \cdot p$ in above equation (let us assign it by $\overline{m}_{phe}$) is the original number of *phe*s which reach the 1st dynode and start the multiplication process in the PMT. So one can re-write finally:

$$\frac{Var(Q)}{\overline{Q}^2} = \frac{1}{\overline{m}_{phe}} \cdot \left(1 + \frac{Var(G)}{\overline{G}^2}\right) \tag{21}$$

Note that if one will plot the measured values of $v(Q)$ for different illumination levels (i.e. for different values of Q) against $\frac{1}{\overline{m}_{phe}}$ (or $\frac{1}{\overline{Q}}$ because $\frac{\overline{Q}}{\overline{m}_{phe}}$ is the searched for conversion factor, i.e. it is a constant) then in the ideal case one will obtain a straight line through the origin of the coordinate system (see Fig.2).

2.3.1 Variable photon detection efficiency p

Let us assume now that the photon detection efficiency p in eq.(16) *does vary depending on to where and under which angle a phe is kicked out.* It is clear that the photocathode of a PMT is not very homogeneous and depending on the impact point of a photon on the photocathode surface area it may exhibit somewhat different efficiencies p_i. The *phe* collection efficiency on the first dynode may show a similar behavior. It is clear that both effects will introduce additional fluctuations in the output charge and thus broaden its amplitude distribution. In this case, unlike eq.(17) one can write:

$$\overline{Q} = \overline{N} \cdot \overline{p} \cdot \overline{G} \tag{22}$$

where $p = \sum p_i \cdot q_i$ (p_i are the individual averages and q_i are respective frequencies in a series of many trials). By differentiating the corresponding probability generation function two times and after some modifications (see for details Breitenberger, 1955) one comes to the result:

$$v(Q) = v(p) + (1 + v(p)) \cdot (v(N) - \frac{1}{\overline{N}}) + \frac{1}{\overline{N} \cdot p} \cdot (1 + v(G)) \tag{23}$$

where $v(p)$ denotes the fractional variance of the distribution of p_i's. If the light source has a Poisson variance then eq.(23) reduces to the form:

$$v(Q) = v(p) + \frac{1}{\overline{N} \cdot \overline{p}} \cdot (1 + v(G)) \tag{24}$$

(this is of course again equation of a straight line but shifted on amount of $v(p)$ in the $v(Q)$ axis, see Fig.2). Note that $v(p)$ is usually much smaller than the second term in eq.(24) and only for very high values of $\overline{N} \cdot \overline{p}$ it becomes not negligible.

2.4 The excess noise factor F

In the theory and the praxis of PMTs one defines the PMT excess noise factor F as the square root from the right bracket in (21):

$$F = \sqrt{(1 + \frac{Var(G)}{\overline{G}^2})} \tag{25}$$

where in fact the $Var(G)$ is the variance of the measured single *phe* distribution and the $\overline{G}$ is its mean. Let us now re-write the eq. (21) for $\overline{m}_{phe}$:

$$\overline{m}_{phe} = \frac{\overline{Q}^2}{Var(Q)} \cdot F^2 \tag{26}$$

Assume one measures a pulse amplitude $\overline{Q}$ at the output of the PMT which is induced by $\overline{m_{phe}} \geq 10$ *phe*s impinging onto the first dynode. One can use then a Gaussian approximation and write $Var(Q) = {\sigma_Q}^2$ and by taking square root from (21) one can write:

$$\frac{\sigma_Q}{\overline{Q}} = \frac{1}{\sqrt{\overline{m}_{phe}}} \cdot F \tag{27}$$

One can see that the left side in the (27) describes the relative width of the charge distribution which in fact is the pulse amplitude resolution. Therefore one can say that the excess noise factor of the PMT is a measure of degradation of its amplitude resolution (making the distribution wider) due to the multiplication process of electrons in the dynode system. For practical purposes sometimes (for fast estimates) it is convenient to use the slightly modified version of the eq. (27) by multiplying both sides with a factor 2.3548 (for a Gaussian distribution the Full Width at Half Maximum of the distribution is equal $FWHM = 2 \cdot \sqrt{2 \cdot ln2} \cdot \sigma$):

$$\frac{FWHM}{\overline{Q}} = 2.3548 \cdot \frac{1}{\sqrt{\overline{m}_{phe}}} \cdot F \tag{28}$$

and

$$\overline{m}_{phe} = (2.3548 \cdot F \cdot \frac{\overline{Q}}{FWHM})^2 = 5.545 \cdot F^2 \cdot (\frac{\overline{Q}}{FWHM})^2 \tag{29}$$

This is the beauty of the method: providing one knows the measured value of the excess noise factor F (with some accuracy) one can calculate the mean number of photoelectrons in the trials (again with some accuracy) by measuring the $FWHM$ and the mean value of the charge distribution at the PMT output. Also, one can obtain the conversion factor from *phe* to digital counts just by taking the ratio $\frac{\overline{Q}}{\overline{m}_{phe}}$.

2.5 Limitations of the method

By comparing the results of measurements with the simulated data in ref. (Wright, a)) it is shown that the eq. (27) is working well for $10 \leq \overline{m}_{phe} \leq 10^5$. It is clear that when the PMT (or some other light sensor with good amplitude resolution) shows more than one peak in its output amplitude distribution at very low levels of illumination as, for example, tested by us hybrid PMTs which show a distribution with multiple peaks for 1-5 *phe*s (see Fig.1), the formula (27) can not be applied. Usually when the initial number of *phe*s is ≥ 10 then the output distribution can be approximated by a Gaussian distribution. For such a distribution one can measure both the mean $\overline{Q}$ and its σ_Q. Obviously the precision will depend on the accuracy of the measured F.

2.6 Error propagation formula for the m_{phe}

Let us now differentiate the eq. (26) in order to obtain the error propagation formula. We will obtain:

$$\frac{\Delta m_{phe}}{m_{phe}} = 2 \cdot \frac{\Delta F}{F} \tag{30}$$

This means that if we want to measure the m_{phe} with 10% accuracy, then the F factor must be measured with a precision of 5% and if we want to obtain an accuracy of 5% for m_{phe} then F must be measured 2.5 % accurate.

2.7 Very small amplitudes in the single phe distribution

In the single *phe* amplitude distribution of a PMT one has to measure carefully the "left" (from the peak) side of the distribution, which lies below the "valley" (see Fig.3). There are lot of "hidden" *phe*s there with very small amplitudes, which are often masked by the pedestal distribution (as one can see from Fig.3 the effect of different tails is proportional to the area below the corresponding curves). The main reason for these very low amplitudes is first of all due to the inelastic back-scattering of *phe*s from the first dynode (the percentage of back-scattered electrons is proportional to the average Z of the used material), which often loose just a tiny portion of their initial energy in the first interaction. Also, the *phe*s which hit the edges of dynodes suffer in the subsequent multiplication due to charge losses being thus an additional source of small amplitudes.

The above mentioned very small amplitudes contribute heavily in the calculation of especially $Var(Q)$ by increasing its value (because it takes the second moments i.e. the charge lying further from the mean is weighted by the squared distance) and also in $\overline{G}$ by lowering its value in formula (25) (note that as a measure of dispersion, $\sqrt{Var(Q)}$ is more sensitive to large deviations than the *mean* of the distribution). As a result with inclusion of smaller and smaller amplitudes the ratio $\frac{Var(G)}{\overline{G}^2}$ will increase (and thus the value of the F factor will increase). Note that the precision of measuring the gain of a PMT $\overline{G}$ (i.e. *the mean of the single phe distribution, which is in fact the phe to digital counts conversion factor*), when one considers only the amplitudes above the valley in the single *phe* amplitude distribution, is usually overestimated by $\sim 20\%$. A very careful measurement can allow one to measure it with an accuracy of -5%, which can be considered as a lower limit (for details see Wright, b)).
In (Mirzoyan & Lorenz, 1997) is shown that by neglecting the small amplitudes below valley in the single *phe* distribution for some PMTs one can make an error of up to 55 % in measuring their absolute gain.

3 Discussion

As it is mentioned in (Mirzoyan & Lorenz, 1997) one can carefully measure the single *phe* response of a PMT and just by calculating its *mean* one can obtain the *phe* to digital counts conversion factor (knowledge of the absolute charge sensitivity of the used ADC provides also the absolute measurement of the PMT gain). This is relatively easy to do in the laboratory for a fixed applied HV and by providing (almost) ideal conditions for suppressing any pick-up noise (any source of electro-magnetic noise will make the pedestal distribution wider thus even more "hiding" the real *phe* events of very small amplitudes). By varying the HV and knowing the gain dependence of the PMT on the HV one can still scale the mean of the distribution. The latter will be difficult to do for the whole imaging camera because one has to know and be able to apply the gain to HV individual behaviors for each of the several hundred channels. The same measurement will be somewhat more difficult to perform "in situ" on the telescope because of lot of different devices generating electro-magnetic noise (I know this from my experience of trying to perform single *phe* measurements at Roque in La Palma). Also the noise of the laser and of the related amplifiers in the chain of analog signal optical transformation line can "screen" the very small amplitudes.
Unlike the above mentioned method one can use the formalism of the F factor discussed in this report. The knowledge of the F factor will allow one to use,

for example, the daily available laser calibration data in order to measure the conversion factors continuously. There are still open questions which should be clarified as, for example, those listed below:

- if the light from the laser has a Poisson variance (see the eq.(19)). One can normalize the laser data and get rid of its intensity fluctuations, but still the question is open.
- how does the F factor behave when varying the HV in large margins (one would expect a very weak dependence although only very careful measurements can answer this question).
- what is the precision of the F factor method (the F factor takes into account only the second moments of the asymmetric single *phe* distribution) compared to that of the method of determination of the *mean of the single phe amplitude distribution.*

The last problem was studied in (Mirzoyan & Lorenz, 1997) but probably more studies will be necessary.
Usually the PMTs show very different behavior of small amplitude tails below the valley, (see three different possible scaenarios on Fig.3 with corresponding *mean values*), even if they are from the same production batch. Because of absence of any general theory explaining these differences the only way is to carefully measure the individual PMTs.

4 Conclusions

The method of measuring PMT specific F factors will allow us to perform *phe* to digital counts calibrations of the MAGIC telescope on everyday basis in a very simple way: one shell just apply this method to the camera flat-fielding (laser and/or LED) data files.

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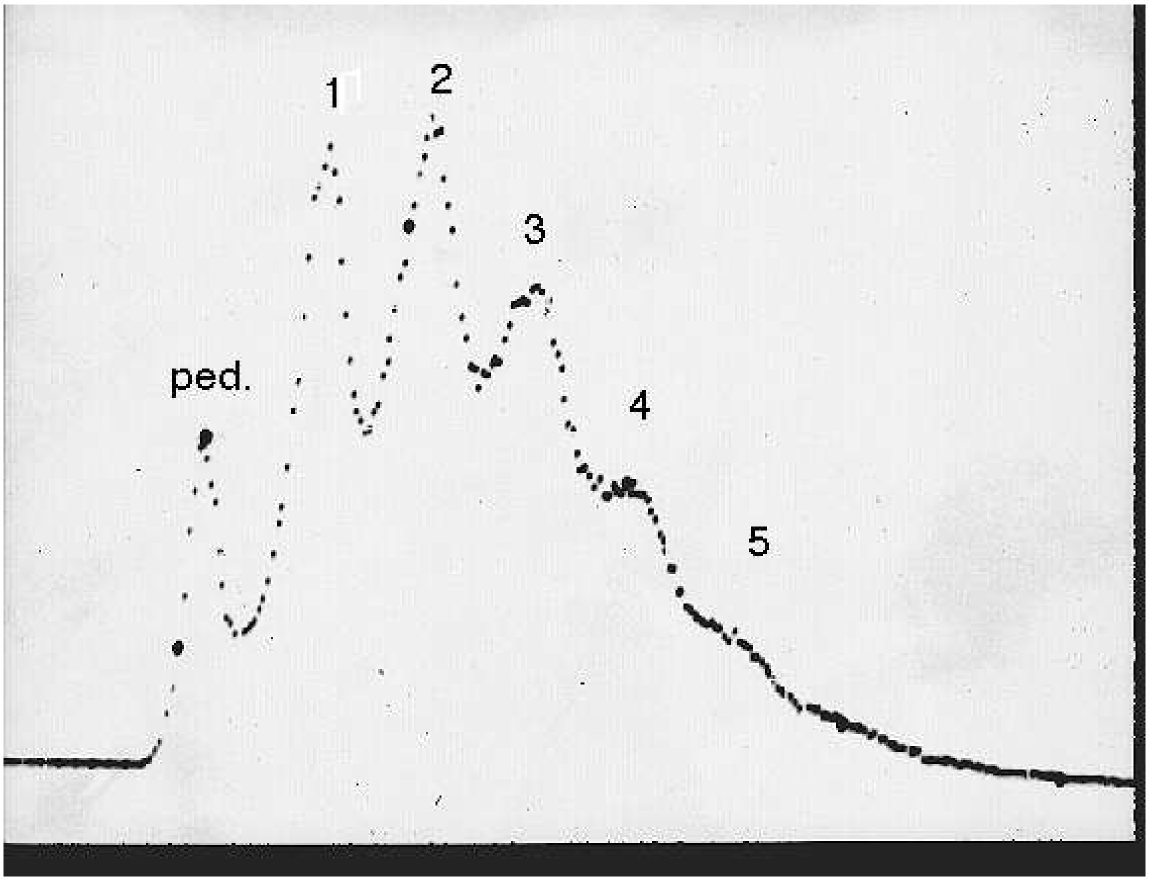

Fig. 1. The amplitude resolution of the R7110U-40 HPD. The light level is adjusted to provide 2.2 *ph.e.*s in the mean. The first peak is the pedestal.

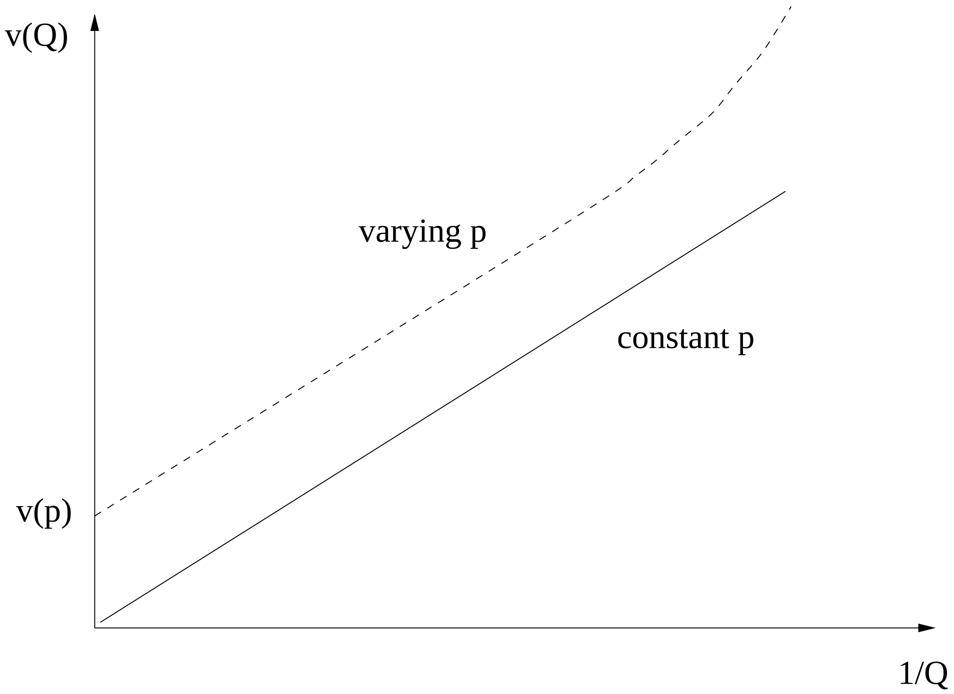

Fig. 2. Dependence of $v(Q)$ on $1/\overline{Q}$ (or similarily, on $1/\overline{m}_{phe}$) for constant and varying p (see eq. (20) and (24) in the text).

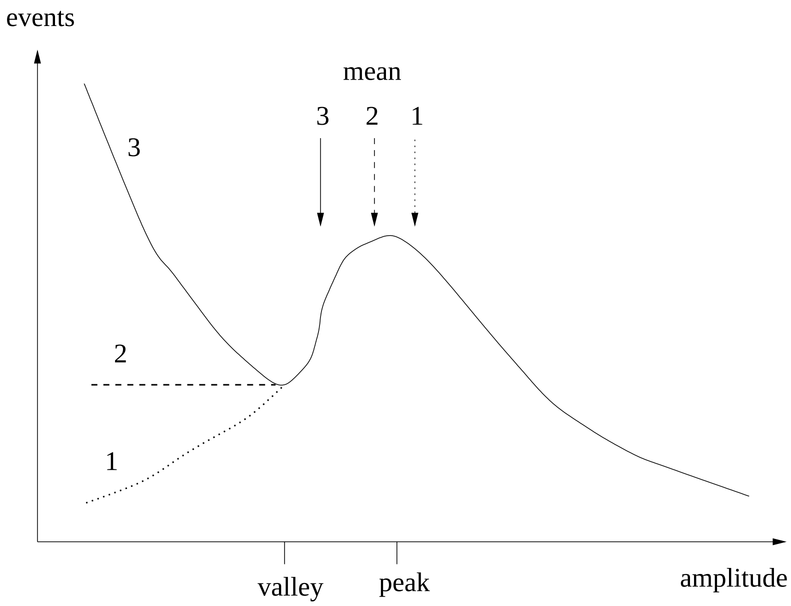

Fig. 3. Three possible scenarios for the behavior of very small amplitudes below the valley and the corresponding *mean*s in the single *phe* amplitude distributions of PMTs.